

Design of Circular Beam Example



ACI 318-99

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Design of Circular Beam Example 1 (ACI 318-99)

Design a semi-circular beam supported on three-equally spaced columns. The centers of the columns are on a circular curve of diameter 8m. The beam is support a uniformly distributed factored load of 5.0 t/m, in addition to its own weight.

Use $f'_c = 350 \text{ kg/cm}^2$ and $f_y = 4200 \text{ kg/cm}^2$

Solution

$$L = r(\pi/2) = 6.28 \text{ m}$$

$$h_{\min} = L / 18.5 = 628 / 18.5 = 33.94 \text{ cm}$$

use Beam 40×70 cm

$$\text{o. w. of the beam} = 0.4(0.7)(2.5)(1.4) = 0.98 \text{ t/m'}$$

$$\text{total load} = 5.14 + 0.84 = 5.98 \text{ t/m'}$$

$$M_{\max(-ve)} = 0.429wR^2 = 0.429(5.98)(4)^2 = 41.05 \text{ t.m}$$

$$M_{\max(+ve)} = 0.1514wR^2 = 0.1514(5.98)(4)^2 = 14.48 \text{ t.m}$$

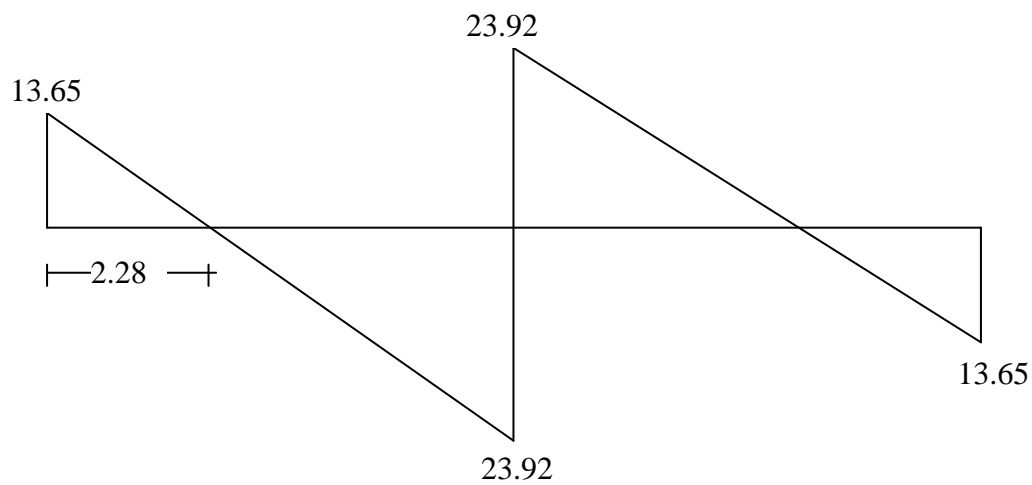
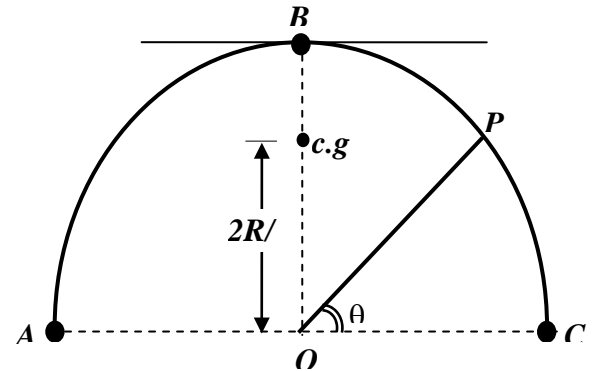
$$T_{\max} = 0.1042wR^2 = 0.1042(5.98)(4)^2 = 9.97 \text{ t.m}$$

Reactions

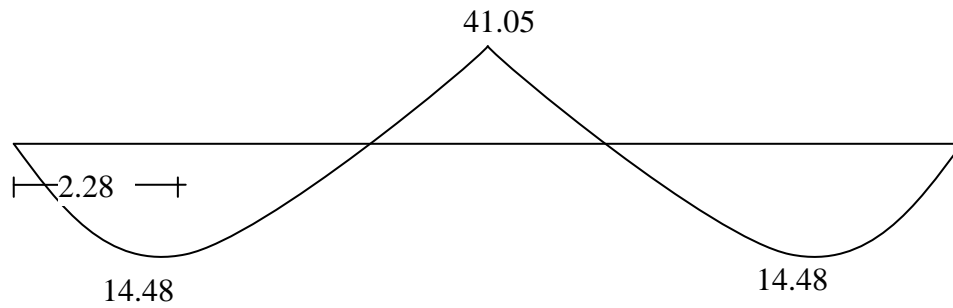
$$R_A = \frac{wR}{2}(\pi - 2) = \frac{5.98(4)}{2}(\pi - 2) = 13.65 \text{ ton}$$

$$R_B = 2wR = 2(5.98)(4) = 47.84 \text{ ton}$$

$$\text{Shear at point P: } V_{\theta} = \frac{wR}{2}(\pi - 2) - wR\theta$$



Shear Force Diagram



Bending Moment Diagram

Design for Reinforcement

$$d = 70 - 4.0 - 0.8 - 1.25 = 63.95$$

$$\rho_{-ve} = \frac{0.85 \cdot 350}{4200} \left[1 - \sqrt{1 - \frac{2.61 \cdot 10^5 (41.05)}{40 \cdot 63.95 \cdot 350}} \right] = 0.00697 > \rho_{\min}$$

$$A_{s(-ve)} = (0.00697)(40)(63.95) = 17.85 \text{ cm}^2$$

$$\rho_{-ve} = \frac{0.85 \cdot 350}{4200} \left[1 - \sqrt{1 - \frac{2.61 \cdot 10^5 (14.48)}{40 \cdot 63.95 \cdot 350}} \right] = 0.00238 < \rho_{\min}$$

$$A_{s(+ve)} = (0.0033)(40)(63.95) = 8.44 \text{ cm}^2$$

Design for Shear

$$V_{\max} = 23.92 \text{ ton} \quad (T = 0.0 \text{ at middle support})$$

$$\phi V_c = 0.53(0.85) \sqrt{350} (40)(63.95) = 21.56 \text{ ton}$$

$$V_s = \frac{23.92 - 21.56}{0.85} = 2.77 \text{ ton}$$

$$2.77 \cdot 1000 = \frac{A_v}{S} \cdot 4200(63.95)$$

$$\frac{A_v}{S} = 0.0103 \text{ cm}^2 / \text{cm}$$

Design for Torsion

$$T_u \leq \frac{0.265\phi\sqrt{f'_c}A_{cp}^2}{p_{cp}} \Rightarrow \text{neglect torsion}$$

$$A_{cp} = b h p_{cp} = 2(b+h), A_{oh} = x_0 y_0, p_h = 2(x_0 + y_0) \text{ and } A_0 = 0.85x_0 y_0$$

The following equation has to be satisfied to check for ductility

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)} \leq 0.263\phi\sqrt{f'_c}$$

Reinforcement

$$\frac{A_T}{S} = \frac{T_u}{2\phi f_{ys} A_0}$$

$$A_l = \left(\frac{A_T}{S}\right)p_h \text{ and } A_{l,\min} = \frac{1.3\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_T}{S}\right)p_h$$

At section of maximum torsion is located at $\theta=59.43$

$$T=9.97 \text{ t.m}$$

$$V_u = \frac{wR}{2}(\pi - 2) - wR\theta = 13.65 - 5.98(4)\left(\frac{59.43}{360}2\pi\right) = -11.6 \text{ ton}$$

$$M = 0.0$$

$$\phi V_c = 0.53(0.85)\sqrt{350}(40)(63.95) = 21.56 \text{ ton} > V_u \text{ (No shear reinf. required)}$$

$$\frac{0.265\phi\sqrt{f'_c}A_{cp}^2}{p_{cp}} = \frac{0.265(0.85)\sqrt{f'_c}(40 \cdot 70)^2}{2(40+70)10^5} = 1.50 \text{ t.m} < T_u$$

i.e torsion must be considered

Ductility Check

The following equation has to be satisfied

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)} \leq 0.263 \phi \sqrt{f'_c}$$

$$x_0 = 40 - 4 - 4 - 0.8 = 31.2$$

$$y_0 = 70 - 4 - 4 - 0.8 = 61.2$$

$$p_h = 2(x_0 + y_0) = 2(31.2 + 61.2) = 184.8 \text{ cm}$$

$$A_{oh} = 31.2 \cdot 61.2 = 1909.44 \text{ cm}^2$$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)} = \sqrt{\left(\frac{11.16 \cdot 10^3}{40 \cdot 63.95}\right)^2 + \left(\frac{(9.97 \cdot 10^5)(184.8)}{1.7(1909.44)^2}\right)} = 30.04 \text{ kg/cm}^2$$

$$0.263 \phi \sqrt{f'_c} = 0.263(0.85) \sqrt{350} = 41.82 \text{ kg/cm}^2 > 30.04 \text{ kg/cm}^2$$

i.e section dimension are adequate for preventing brittle failure due to combined shear stresses.

$$\frac{A_T}{S} = \frac{T_u}{2 \phi f_{ys} A_0} = \frac{9.97 \cdot 10^5}{2(0.85) 4200(0.85 \cdot 1909.44)} = 0.086 \text{ cm}^2 / \text{cm}$$

$$\left(\frac{A_T}{S}\right)_{\min} = \frac{3.5 b_w}{2 f_{ys}} = \frac{3.5(40)}{2(4200)} = 0.0167 \text{ cm}^2 / \text{cm}$$

$$\left(\frac{A_V}{S}\right) + \left(\frac{A_T}{S}\right) = 0 + 0.086 = 0.086 \text{ cm}^2 / \text{cm}$$

Use $\phi 12\text{mm} @ 12.5\text{cm}$ (closed stirrups) with $\frac{A}{S} = \frac{1.13}{12.5} = 0.0904 \text{ cm}^2 / \text{cm}$

$$A_l = \left(\frac{A_T}{S}\right) p_h = 0.086(184.8) = 15.98 \text{ cm}^2$$

$$A_{l,\min} = \frac{1.3 \sqrt{f'_c} A_{cp}}{f_y} - \left(\frac{A_T}{S}\right) p_h = \frac{1.3 \sqrt{350} (40 \cdot 70)}{4200} - 15.98 = 0.234 \text{ cm}^2$$

$$A_l = \frac{15.98}{3} = 5.33 \text{ cm}^2$$

Use 2 $\phi 20$ top and 4 $\phi 14$ skin reinforcement

$$(A_{s,+ve})_{\text{total}} = 5.33 + 8.44 = 13.77 \text{ cm}^2 \quad \text{use } 6 \phi 18\text{mm}$$

$$(A_{s,-ve})_{\text{total}} = 17.83 + 8.44 = 23.16 \text{ cm}^2 \quad \text{use } 8 \phi 20\text{mm}$$

