# การอบรมก่อนทดสอบความรู้ทางวิศวกรรม

## ระดับภาคีวิศวกร สาขาวิศวกรรมโยธา

**วิชา Structural Analysis**


รองศาสตราจารย์สิริวัฒน์ ไชยชนะ

**ผู้บรรยาย**


28 ธันวาคม 2551

# <u>รายละเอียดวิชาการทดสอบความรู้ทางวิศวกรรม</u>
# <u>ระดับภาคีวิศวกร</u>

- <u>สาขาวิศวกรรมโยธา</u>

1. วิชา **Structural Analysis**

Analysis of indeterminate structures by elastic load method, methods of slope and deflection, moment distribution, strain energy; influence line of indeterminate structures; introduction to plastic analysis; approximate analysis; introduction to matrix structural analysis.

**โครงสร้าง**

รูปทรงหรือรูปร่างที่ประกอบขึ้นจากชิ้นส่วนหรือองค์อาคาร(members) มากมายหลายชิ้นส่วนมาเชื่อมยึดต่อกันอย่างต่อเนื่อง เพื่อผลต่อการเกิดความสามารถต้านทานน้ำหนักหรือแรงกระทำในทิศทางต่างๆได้ โดยไม่เกิดการเปลี่ยนรูปทรงที่ชัดเจนและถาวรกับส่วนใดส่วนหนึ่งของขิ้นส่วนเหล่านั้น

**ประเภทของโครงสร้าง**

**FRAME STRUCTURE (แบบโครงข้อ)**

Skeleton System

*อาคารทั่วไป*

**MASS STRUCTURE (แบบมวล)**

Self Wt.

*เขื่อน*

**ประเภทของโครงข้อ**

***Beam - Column***

***Frames***

***Plane Truss***

***Space Truss***

# เราวิเคราะห์โครงสร้างไปทำไม?

จากคำจำกัดความของโครงสร้างที่กล่าวไว้แล้วว่าเป็นรูปทรงหรือรูปร่างที่ประกอบขึ้นจากชิ้นส่วนหรือองค์อาคาร (members)มากมายหลายชิ้นส่วนมาเชื่อมยึดต่อกันอย่างต่อเนื่อง เพื่อผลต่อการเกิดความสามารถต้านทานน้ำหนักหรือแรงกระทำในทิศทางต่างๆได้ โดยไม่เกิดการเปลี่ยนรูปทรงที่ชัดเจนและถาวรกับส่วนใดส่วนหนึ่งของชิ้นส่วนเหล่านั้น

***ดังนั้น เพื่อให้โครงสร้างรับแรงกระทำได้ และไม่เปลี่ยนรูป จึงต้องวิเคราะห์ดูว่าเมื่อมีแรงมากระทำ จะเกิดผลลัพธ์ขึ้นกับชิ้นส่วนอย่างไร จะหดหรือยืด จะแอ่นหรือโก่งงอพับอย่างไรบ้าง เพื่อที่จะสรุปว่าแต่ละผลที่จะเกิดขึ้นนั้นควรจะเลือกชนิดวัสดุ เลือกขนาดหรือรูปแบบของหน้าตัดที่เหมาะสมเพียงไรมาเป็นชิ้นส่วนประกอบกันเป็นโครงสร้าง***

# หลักการวิเคราะห์โครงสร้าง

**กฎแห่งการสมดุลของชิ้นส่วนโครงสร้าง**

**Equilibrium Conditions**

External Forces = Internal Forces

External Forces = External Forces
*(Action)* *(Reaction)*

**Equilibrium Equations**

$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$

$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$

# ลำดับการวิเคราะห์โครงสร้าง

**ชนิดและรูปแบบของแรงกระทำต่อการเลือกประเภทของโครงสร้าง**

Vertical Forces — *Beams , Trusses*

Horizontal Forces — *Frames*

**ผลการตอบสนองต่อแรงของชิ้นส่วน**

Shear Forces
Bending Moment
Deformation

**เครื่องมือและวิธีวิเคราะห์**

**STATICALLY DETERMINATE STRUCTURES** ➡ *Unknown* $\leq 3$ ➡ *Use Only EE*

**STATICALLY INDETERMINATE STRUCTURES** ➡ *Unknown* $> 3$

➡ *Call The unknowns exceed 3 ,'Redundant'*

➡ *Degree of indeterminacy are amount of redundant*

# เครื่องมือและวิธีวิเคราะห์

| | **Internal Forces<br>(Shear, Bending Moment, Reactions)** | **Deformation<br>(Rotation, Deflection, Drift)** |
|---|---|---|
| SDS | EE, $\sum F_x = 0$ $\sum F_y = 0$ $\sum M = 0$ | McCaulay's Method<br>Moment-area Method<br>Conjugate-beam Method<br>Virtual work Method<br>Castigliano's Theorem |
| SIS<br><br><br>*With EE* | Consistent Deformation Method<br>Elastic Load Method, Strain Energy Method<br>Theorem of Least Work, Column Analogy<br>Three Moments Equation Method<br>Slope-Deflection Method<br>Moment Distribution Method<br>Approximate Analysis → *For Horizontal Forces*<br>Plastic Analysis Method<br>Matrix Method | Same as SDS<br>*Force Method*<br><br><br>*Displacement Method* |

# TRUSSES

**Type of Trusses**

- *Plane Truss*
  - *Roof Truss*
  - *Bridge Truss*
- *Space Truss*

**Assumption**

1. *ให้แนวแกนของปลายชิ้นส่วนที่พบกันที่จุดต่อ จะต้องตัดหรือบรรจบกันที่จุดๆเดียว*
2. *จุดต่อจะต้องไม่มีแรงฝืดหรือแรงเสียดทาน*
3. *แรงกระทำภายนอก กำหนดให้ต้องกระทำที่จุดต่อเท่านั้น*

**Stability and Determinacy(or Indeterminacy) Conditions**

**Stability Condition:** $M \geq 2j - 3$

**Determinacy Condition:** $M = 2j - r \longrightarrow M = 2j - 3$

**Indeterminacy Condition:** $i = M - (2j - r)$

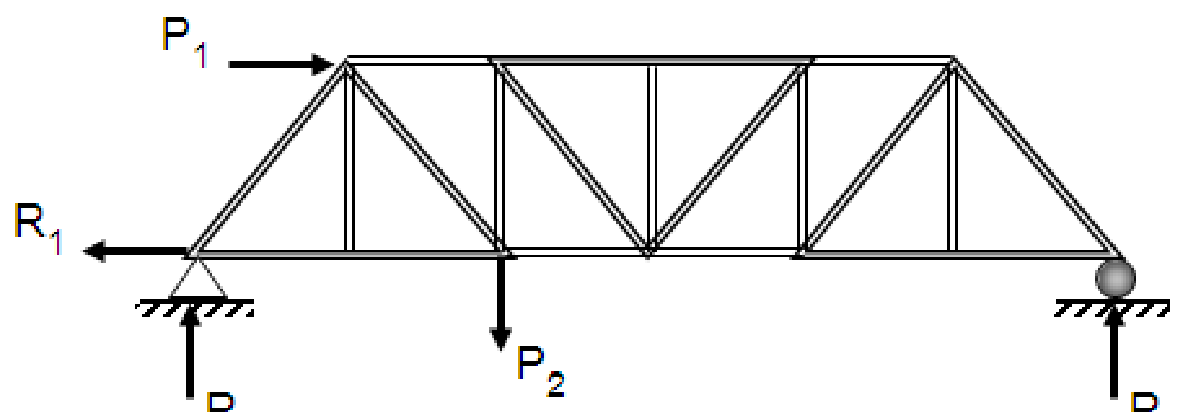
สรุป

- $M < 2j - 3 \longrightarrow$ โครงสร้างไม่มั่นคง
- $M = 2j - 3 \longrightarrow$ โครงสร้างมั่นคงและเป็นดิเทอร์มิเนท
- $M > 2j - 3 \longrightarrow$ โครงสร้างมั่นคงเกินและเป็นอินดิเทอร์มิเนท

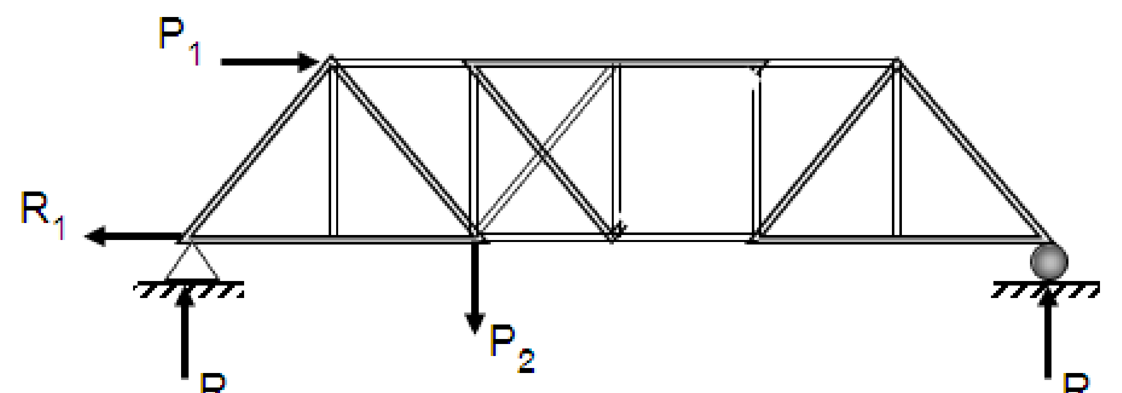
**Method for Analysis Indeterminate Truss**

**Unit Load Method for Singly-Redundant Truss**

**Theorem of Least Work**




(ก) $r = 3$ ,$m = 21$ ,$j = 12$ ,$m = 2j - r$ , มั่นคง

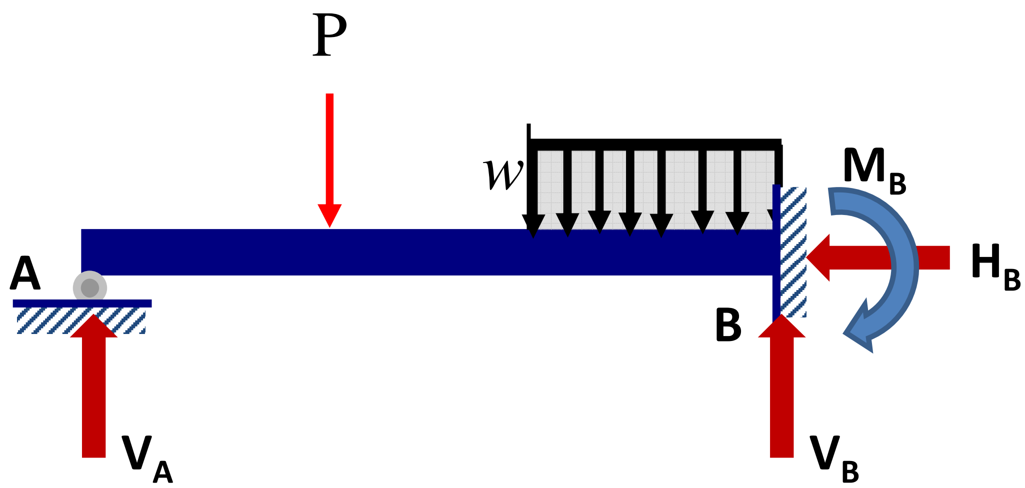



(ค) $r = 3$ ,$m = 21$ ,$j = 13$ ,$m < 2j - r$ , ไม่มั่นคง

# BEAMS

## Method of Least Work

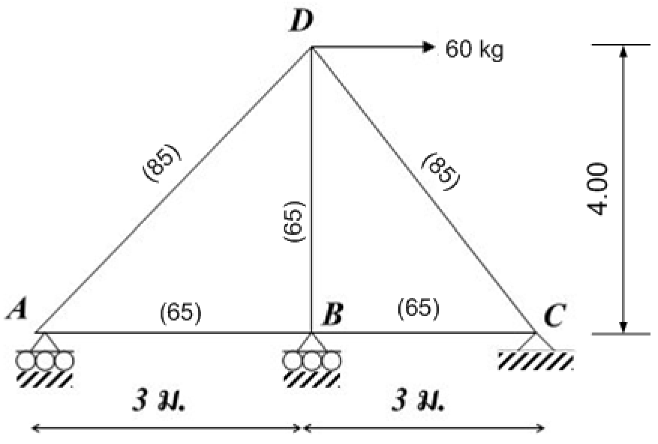
**Assumption:** โครงสร้างใดๆที่ไม่มีการขยับตัวของที่รองรับ และไม่มีการเปลี่ยนรูปเนื่องจากอุณหภูมิเกี่ยวข้อง ตัวไม่ทราบค่าส่วนเกิน(**Redundant**) จะทำให้พลังงานสะสมภายใน(**The internal Energy**) มีค่าน้อยลงจนถึงต่ำสุด



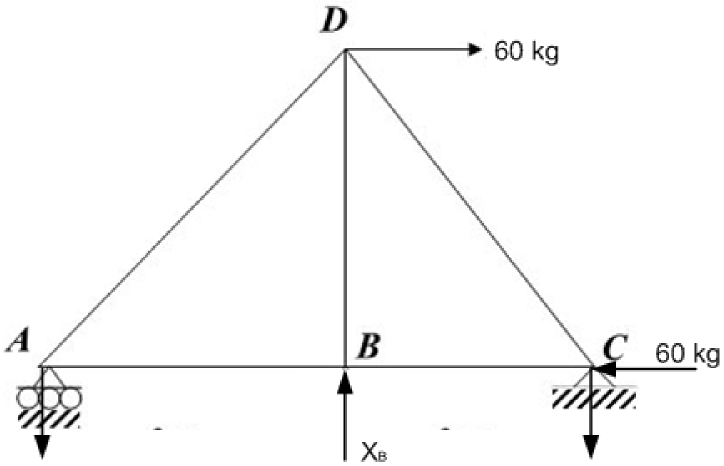

*From Castigliano's Theorem* $\Delta_A = \partial W/\partial P_A$

*ในที่นี้* $P_A = V_A$ → $\Delta_A = \partial W/\partial V_A = 0$

From Assumption




$E = 2 \times 10^6$ ksc




Let $X_B$ as redundant

From Theorem of least work

$$\delta_B = \partial U/\partial X_B = 0$$

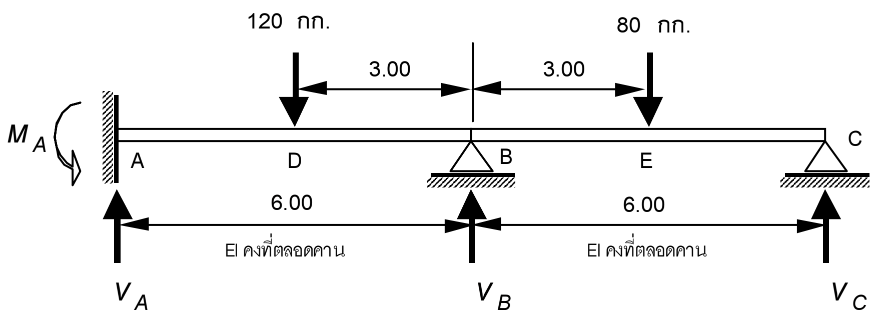
$$\sum NL/AE\, \partial N/\partial X_1 = 0$$

| member | L | A | N | $\partial N/\partial X_B$ | $NL/AE\, \partial N/\partial X_B$ |
|---|---|---|---|---|---|
| BC | 6 | 60 | +20 | 0 | 0 |
| CF | 5 | 50 | -100/3 | 0 | 0 |
| BF | 5 | 50 | +5/3(20-X1) | -5/3 | -25/90(20-X1) |
| CE | 4 | 40 | +4/3(X1) | +4/3 | +16/90(X1) |
| FE | 3 | 30 | +40 | 0 | 0 |
| FD | 5 | 50 | -5/3(X1) | -5/3 | +25/90(X1) |
| ED | 4 | 40 | +4/3(X1) | +4/3 | +16/90(X1) |
| | | | | $\sum$ | -500+82(X1) |

$$\frac{104+1203X_B}{E} = 0$$

$$X_B = -8.65 \text{ kg.}$$

***จงวิเคราะห์คานแบบอินดิเทอร์มิเนท ดังแสดงในรูป***




1.คานที่กำหนดให้เป็นแบบอินดิเทอร์มิเนทขีดขนาดเท่ากับ 2

2.เลือก $V_C$ กับ $V_B$ เป็น redundant

3.ใช้หลักการสมดุลที่หน้าตัด เขียนสมการของโมเมนต์ ดังนี้

ช่วง CE;ค่า x เริ่มที่ C จะได้ $M = V_C x$

ช่วง EB;ค่า x เริ่มที่ E จะได้ $M = V_C(3 + x) - 80x$

ช่วงBD;ค่า x เริ่มที่ B จะได้ $M = V_C(6 + x) - 80(3 + x) + V_B x$

ช่วงDA;ค่า xเริ่มที่ D จะได้ $M = V_C(9 + x) - 80(6 + x) + V_B(3 + x) - 120x$

4. Partial derivative สมการข้างบน โดยเทียบกับ $V_B$ และ $V_C$

$M = V_C x$ $dM/dV_C = x$ $dM/dV_B = 0$

5. Use of Castigliano's theorem

$dM/dV_C = 0$

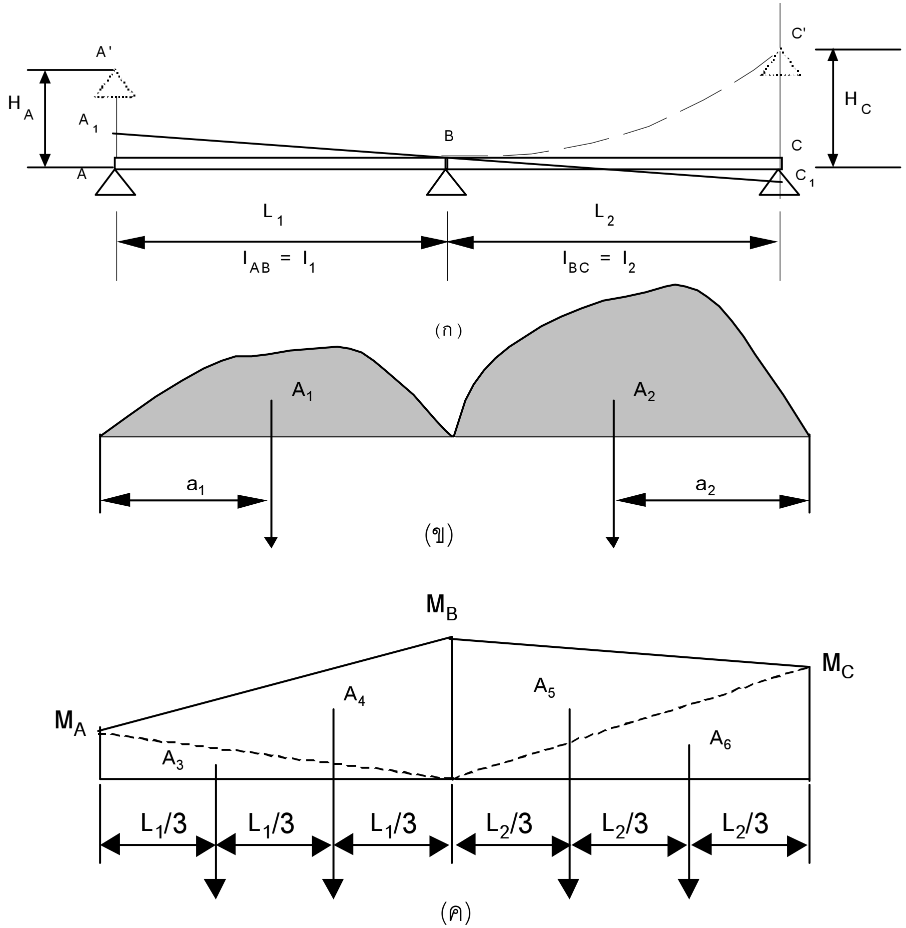
$dM/dV_B = 0$

6. แก้สมการ 2 ชั้น จะได้ค่า $V_B$ และ $V_C$

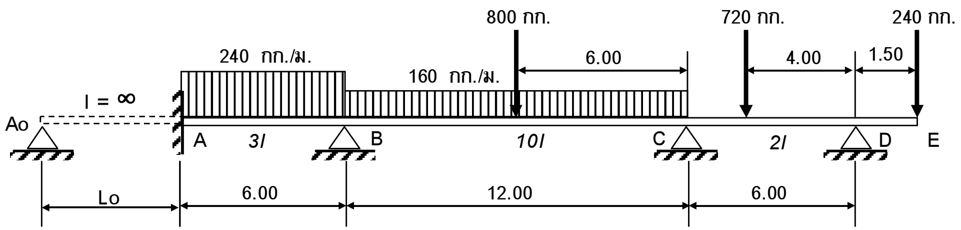
7. แก้สมการสมดุลย์ หาแรงที่เหลือ $V_A$ และ $M_A$

# Method of Three – Moment Equation

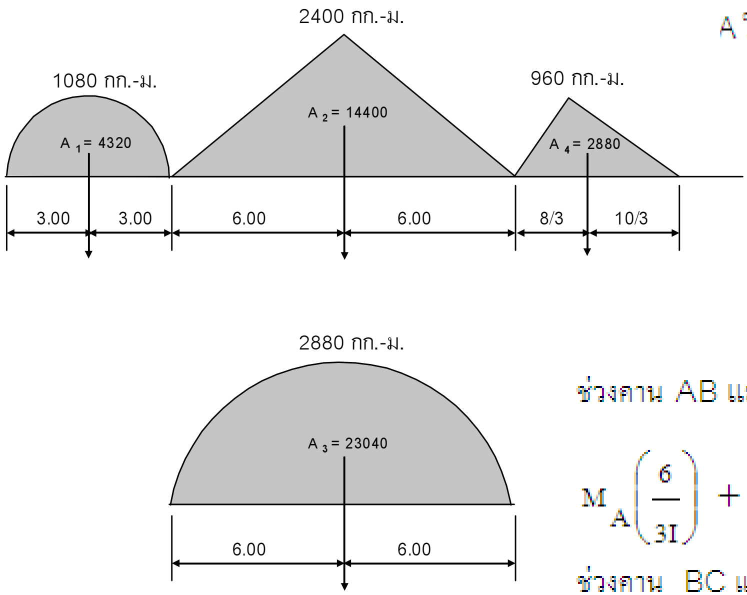
สมการสามโมเมนต์ เป็นความสัมพันธ์ของโมเมนต์ที่จุดรองรับสามตัวของคานต่อเนื่องสองช่วง พิสูจน์ได้จากพื้นฐานของ *ความต่อเนื่อง* ของส่วนโค้งอีลาสติกที่ปลายขวาของคานช่วงซ้ายมือ จะเท่ากับมุมลาดของส่วนโค้งอีลาสติกที่ปลายซ้ายของคานช่วงขวามือ



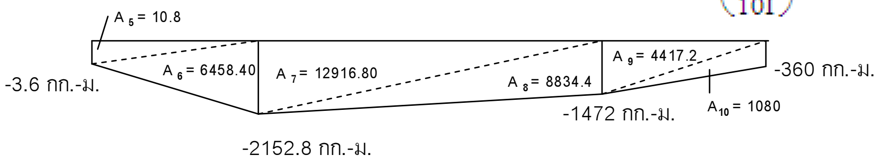

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_C}{L_2}$$




(ก) คานตามกำหนด




(ข) ผังโมเมนต์ดัดในช่วงคาน




(ค) ผังโมเมนต์ดัด ณ จุดรองรับ

1 เนื่องจากที่รองรับ A เป็นแบบยึดแน่น จึงจำเป็นต้องเพิ่มช่วงคานจินตนาการ $A_oA$ ต่อจากจุด A ให้มีความยาว $L_o$ และมีค่าโมเมนต์อินเนอร์เชีย $I = \alpha$

2 ผลจากการตรวจสอบเบื้องต้นได้ว่า $M_{Ao} = 0$ และ $M_D = -360$ กก.-ม.

3 ช่วงคาน $A_oA$ และ AB

$$M_{Ao}\left(\frac{L_o}{\alpha}\right) + 2M_A\left(\frac{L_o}{\alpha} + \frac{6}{3I}\right) + M_B\left(\frac{6}{3I}\right) = -\frac{6(4,320)(3)}{6(3I)}$$

ช่วงคาน AB และ BC

$$M_A\left(\frac{6}{3I}\right) + 2M_B\left(\frac{6}{3I} + \frac{12}{10I}\right) + M_C\left(\frac{12}{10I}\right) = -\frac{6(4,320)(3)}{6(3I)} - \frac{6(14,400)(6)}{12(10I)} - \frac{6(23,040)(6)}{12(10I)}$$

ช่วงคาน BC และ CD

$$M_B\left(\frac{12}{10I}\right) + 2M_C\left(\frac{12}{10I} + \frac{6}{2I}\right) + M_D\left(\frac{6}{2I}\right) = -\frac{6(14,400)(6)}{12(10I)} - \frac{6(23,040)(6)}{12(10I)} - \frac{6(2,880)(10/3)}{6(2I)}$$

4 แก้สมการทั้งสาม จะได้

$M_A$ = -3.60 กก.-ม.

$M_B$ = -2,152.8 กก.-ม.

$M_C$ = -1,472.4 กก.-ม.

# Slope-Deflection Method

**Assumption:** กำหนดให้การเปลี่ยนรูปของโครงสร้างที่จุดต่อเป็นตัวไม่ทราบค่า และตั้งสมมุติฐานไว้ว่าทุกจุดต่อของโครงสร้าง จะต้อง**rigid** เพียงพอที่จะตรึงให้มุมลาดของชิ้นส่วนที่ต่อเนื่องกันไม่เปลี่ยนค่า

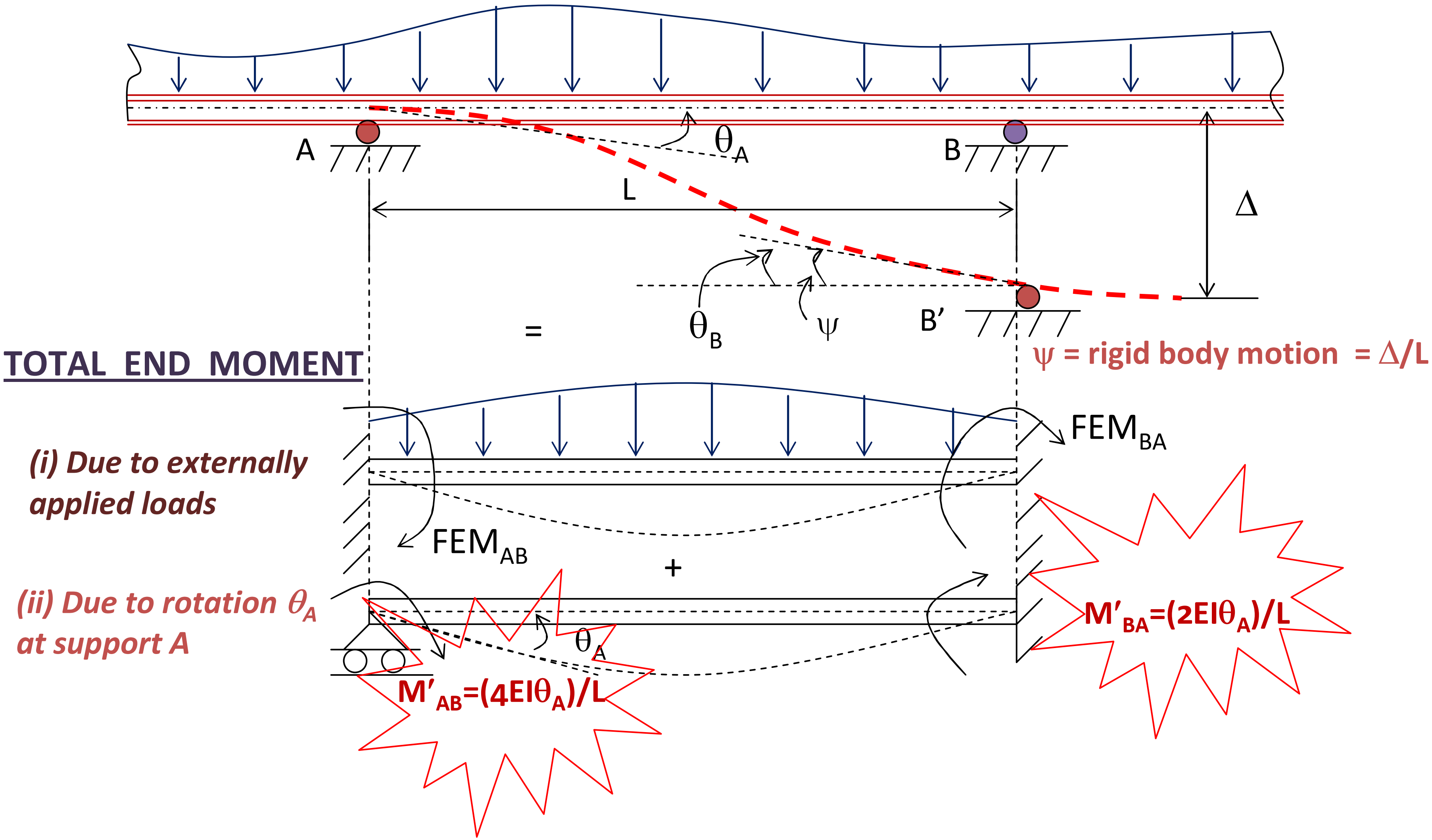
As mentioned earlier, **it is a displacement based analysis for indeterminate structures** - Unknown **displacements** are first **written in terms of the loads by using load-displacement relationships;** then **these equations are solved for the displacements.** Once the displacements are obtained, **unknown loads are determined from the compatibility equations using load-displacement relationships.**

**- Nodes:** Specified points on the structure that undergo displacements (and rotations)

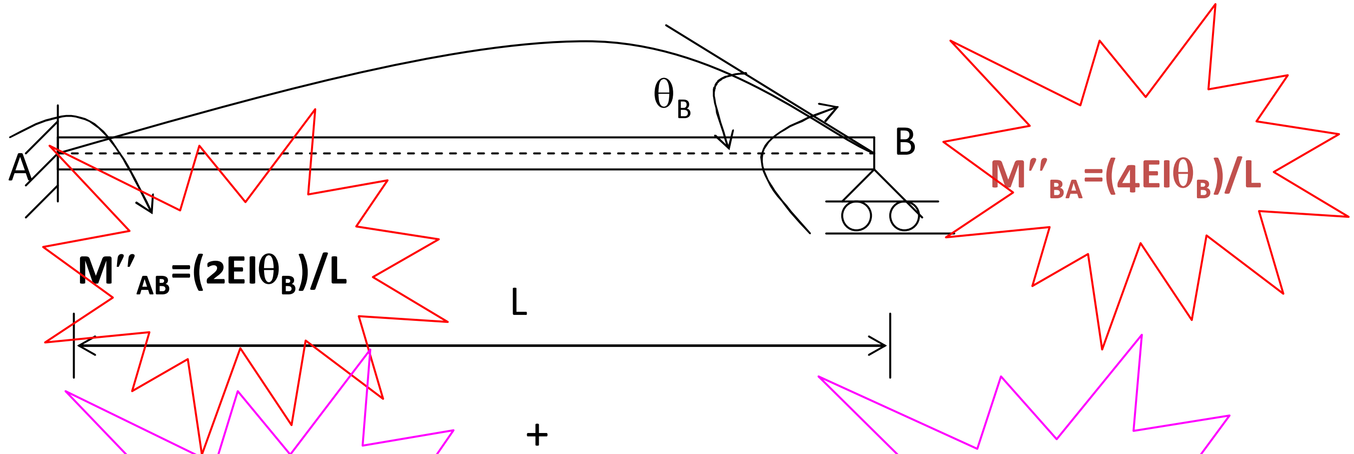
**- Degrees of Freedom:** These displacements (and rotations) are referred to as degrees of freedom

# Slope Deflection Equation

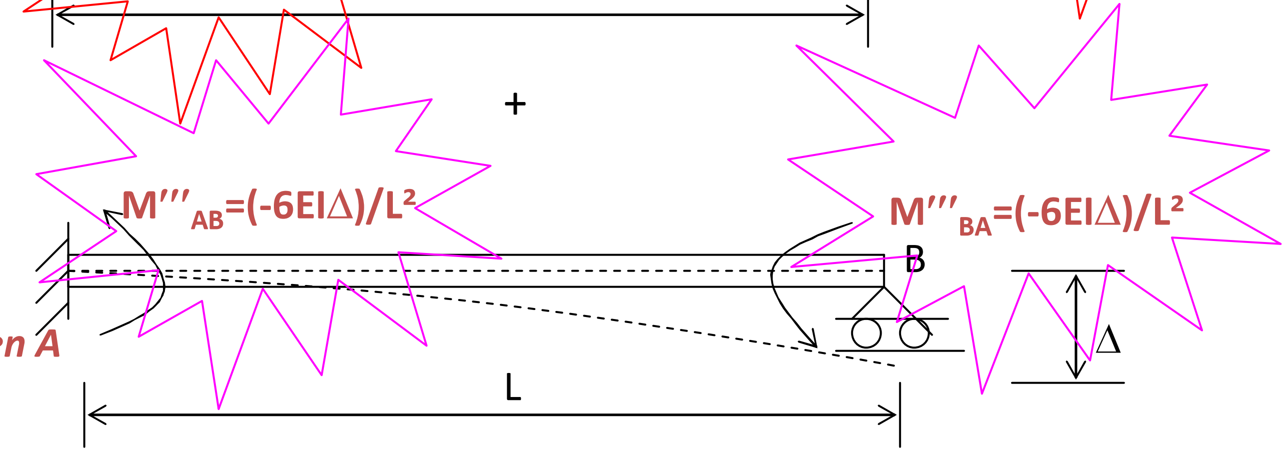
Consider portion AB of a continuous beam, shown below, subjected to a distributed load w(x) per unit length and a support settlement of $\Delta$ at B; EI of the beam is constant.



*(iii) Due to rotation $\theta_B$ at support B*

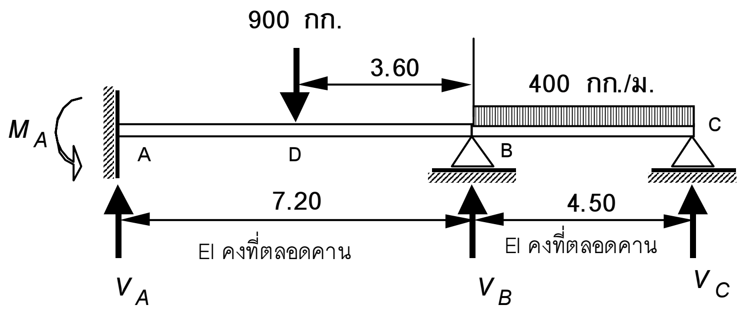



*(iv) Due to differential settlement of $\Delta$ (between A and B)*




$$M_{AB} = (FEM)_{AB} + M'_{AB} + M''_{AB} + M'''_{AB} = (FEM)_{AB} + \frac{4EI\theta_A}{L} + \frac{2EI\theta_B}{L} - \frac{6EI\Delta}{L^2} = \boxed{\frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right) + (FEM)_{AB}} \qquad 4.4.1(\text{ก})$$

$$M_{BA} = (FEM)_{BA} + M'_{BA} + M''_{BA} + M'''_{BA} = (FEM)_{BA} + \frac{2EI\theta_A}{L} + \frac{4EI\theta_B}{L} - \frac{6EI\Delta}{L^2} = \boxed{\frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\Delta}{L}\right) + (FEM)_{BA}} \qquad 4.4.1(\text{ข})$$




1 จากการตรวจสอบเบื้องต้น พบว่าคานต่อเนื่อง ABC ที่วิเคราะห์ มีตัวไม่ทราบค่าที่จุดต่อเพียงสองตัวเท่านั้นคือ $\theta_B$ และ $\theta_C$ เพราะ $\theta_A = 0$ เนื่องจากปลาย A เป็นที่รองรับแบบยึดแน่น ขณะเดียวกัน $\Delta_{AB} = \Delta_{BC} = 0$ เพราะไม่มีการทรุดตัวของที่รองรับเช่นกัน

2 ค่า FEM จากแรงกระทำแต่ละช่วงคาน ดังนี้

$$FEM_{AB} = -\frac{PL}{8} = -\frac{900(7.2)}{8} = -810 \quad \text{กก.-ม.}$$

$$FEM_{BA} = +\frac{PL}{8} = +810 \quad \text{กก.-ม.}$$

$$FEM_{BA} = -\frac{WL^2}{12} = -\frac{400(4.5)^2}{12} = -675 \quad \text{กก.-ม.}$$

$$FEM_{CB} = +\frac{WL^2}{12} = +675 \quad \text{กก.-ม.}$$

3 พิจารณาการสมดุลย์ที่จุดต่อ

จุดต่อB; $M_{BA} + M_{BC} = 0$ (ก)

จุดต่อC; $M_{CB} = 0$ (ข)

4 เมื่อแทนค่าต่าง ๆ ลงในสมการของ SLOPE - DEFLECTION จะได้

$$M_{AB} = -810 + \frac{2EI}{7.2}(\theta_B)$$

$$M_{BA} = 810 + \frac{2EI}{7.2}(2\theta_B)$$

$$M_{BC} = -675 + \frac{2EI}{4.5}(2\theta_B + \theta_C)$$

$$M_{CB} = 675 + \frac{2EI}{4.5}(\theta_B + 2\theta_C)$$

5

$$\theta_B = 165.23 / EI$$

$$\theta_C = -842.75 / EI$$

6 จากนั้นนำค่า $\theta_B$ และ $\theta_C$ ย้อนกลับไปแทนลงในสมการ SLOPE - DEFLECTION อีกครั้ง จะได้

$$M_{AB} = -810 + \frac{2EI}{7.2}\left(\frac{165.23}{EI}\right) = -764.10 \quad \text{กก.-ม.}$$

$$M_{BA} = 810 + \frac{2EI}{7.2}(2)\left(\frac{165.23}{EI}\right) = 902.23 \quad \text{กก.-ม.}$$

$$M_{BC} = -675 + \frac{2EI}{4.5}\left[\frac{2(165.23)}{EI} - \frac{842.75}{EI}\right] = -902.23 \quad \text{กก.-ม.}$$

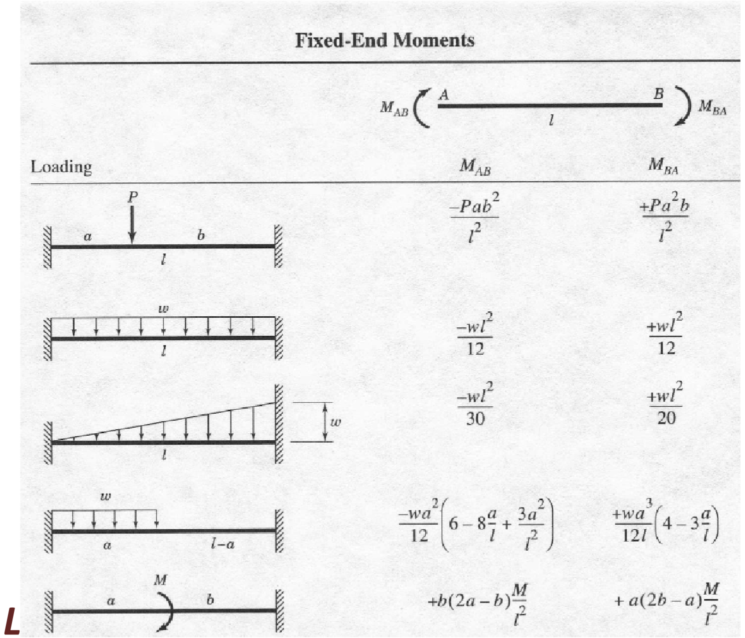
$$M_{BC} = 675 + \frac{2EI}{4.5}\left[\frac{(165.23)}{EI} - \frac{2(842.75)}{EI}\right] = 0 \quad \text{กก.-ม.}$$

# Moment Distribution Method

**Assumption:** พัฒนามาจากวิธี **Slope Deflection** คือกำหนดให้การเปลี่ยนรูปของโครงสร้างที่จุดต่อเป็นตัวไม่ทราบค่า แต่การหาค่าองค์ประกอบของโมเมนต์ปลายต่าง ๆถูกเปลี่ยนไปอยู่ในรูปของแฟคเตอร์ และปล่อยให้มันปรากฏออกมาทีละรอบ ๆ โดยหลักการ **“ล็อคแน่น คลายล็อคและล็อคแน่นอีก”** จนกว่าจะได้ค่าที่ยอมรับได้และถูกต้องมากที่สุด

***STEPS FOR MOMENT DISTRIBUTION***

*(i) Due to externally applied loads , with all joint rotations restrained , Find FEM*


Fixed-End Moments

| Loading | $M_{AB}$ | $M_{BA}$ |
|---|---|---|
| Point load $P$ ($a$, $b$, $l$) | $\frac{-Pab^2}{l^2}$ | $\frac{+Pa^2b}{l^2}$ |
| Uniform load $w$ ($l$) | $\frac{-wl^2}{12}$ | $\frac{+wl^2}{12}$ |
| Triangular load $w$ ($l$) | $\frac{-wl^2}{30}$ | $\frac{+wl^2}{20}$ |
| Partial uniform load $w$ ($a$, $l-a$) | $\frac{-wa^2}{12}\left(6-8\frac{a}{l}+\frac{3a^2}{l^2}\right)$ | $\frac{+wa^3}{12l}\left(4-3\frac{a}{l}\right)$ |
| Moment $M$ ($a$, $b$) | $+b(2a-b)\frac{M}{l^2}$ | $+a(2b-a)\frac{M}{l^2}$ |

*(ii) Relative rotational stiffness :*

*Stiffness Factors (S) : 4EI / L*

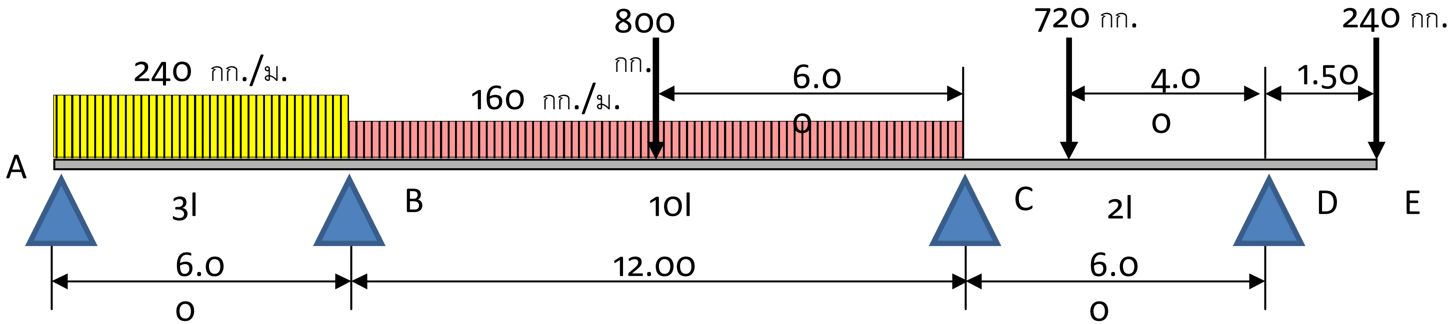
*Carry Over Factor : + 0.5*

*Distribution Factors :* $D_{BA} = (S_{BA})M / \sum S_{Bi}$

*(iii) Distribute the balancing moment :*

*As in table*

# Example




| | 0.5 | | 0.5 | | 0.5 | | |
|---|---|---|---|---|---|---|---|
| **DF** | 1 | 0.375 | 0.625 | 0.714 | 0.286 | 1 | - |
| **FEM** | -720 | +720 | -3120 | +3120 | -640 | +320 | -360 |
| ***Locking Moment*** | +720 | +2400 | | -2480 | | +40 | |
| **Bal. M** | +720 | +900 | +1500 | -1770 | -709 | +40 | |
| **CO. M** | +450 | +360 | -885 | +750 | +20 | -354 | |
| ***Locking Moment*** | -450 | +525 | | -770 | | +354 | |
| **Bal. M** | -450 | -197 | +328 | -550 | -220 | +354 | |
| **CO. M** | +99 | -225 | -275 | +164 | +177 | -110 | |
| ***Locking Moment*** | -99 | +500 | | -341 | | +110 | |
| **Bal. M** | -99 | +187 | +313 | -244 | -97 | +110 | |
| **CO. M** | +94 | -49 | -122 | +156 | +55 | -49 | |
| ***Locking Moment*** | -94 | +171 | | -211 | | +49 | |
| **Bal. M** | -94 | +64 | +107 | -151 | -60 | +49 | |
| ***Total end moments*** | 0 | +2154 | -2154 | +1475 | -1474 | +36 | -360 |

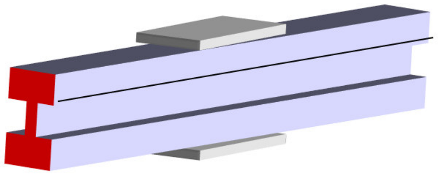
# Column Analogy Method

*“วิธีการของเสาเสมือน เป็นการเปรียบเทียบผลลัพธ์ทางคณิตศาสตร์ ระหว่าง ค่าโมเมนต์ดัดในคานกับค่าความเค้นบนหน้าตัดเสา ที่เกิดเนื่องจากแรงกระทำเยื้องศูนย์”*

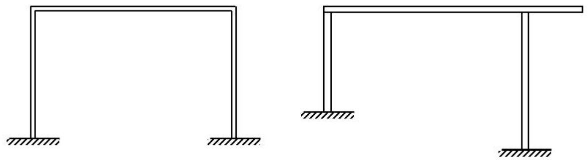
Advantages :

1. Useful in determining FEM for beam element with *constant* or *variable* moment of inertia.
2. Useful in complete analysis of *symmetrical* or *unsymmetrical* rigid frames, either two fixed supports or one closed cell.

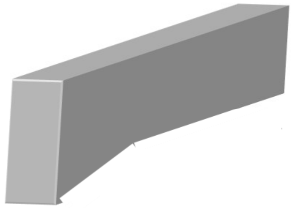
## Suitable Type of Structure



WF beam w/cover plates

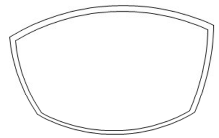


Frames

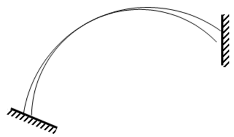


Haunched beam

Box Girder

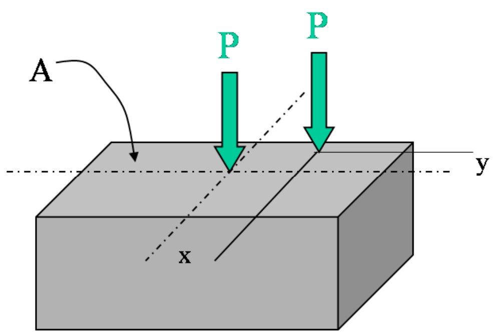


Closed cell structure



Unsymetrical Frames

# Analogous Column [1]




$$P = \int s \, dA$$

$$Py = \int s \, dA \, y$$

$$Px = \int s \, dA \, x$$

$$\text{Combined Stress} = \frac{P}{A} \pm \frac{Mx}{Iy} \pm \frac{My}{Ix}$$

## Conceptual

1. Ms/EI diagram → Primary Structure

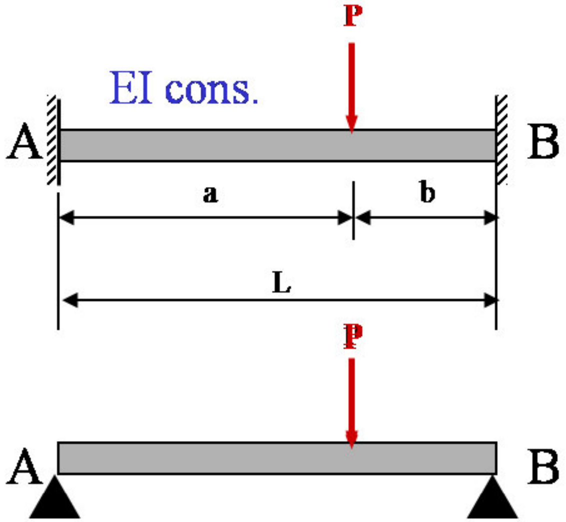
positive

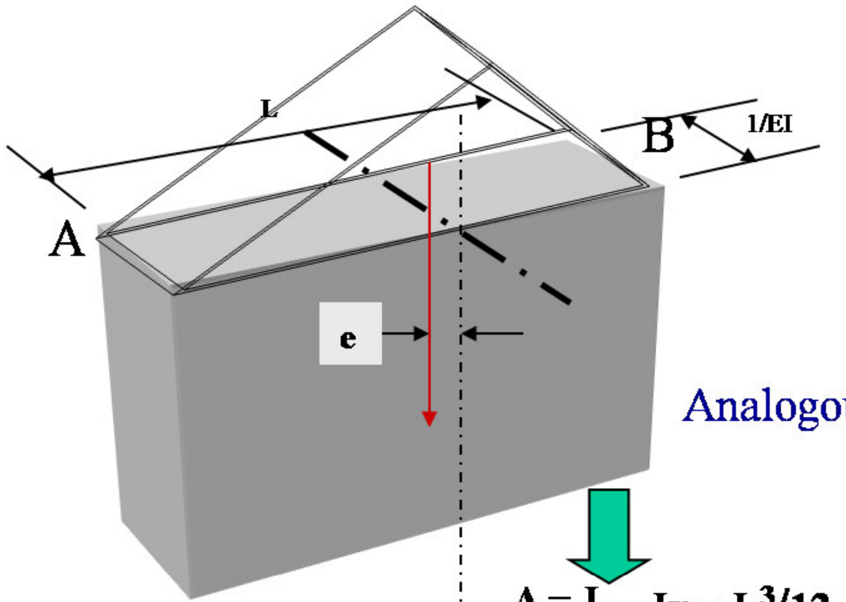
negative

2. Mi(internal) → Analogous Column

3. $\boxed{M = Ms - Mi}$ → Analysis Structure

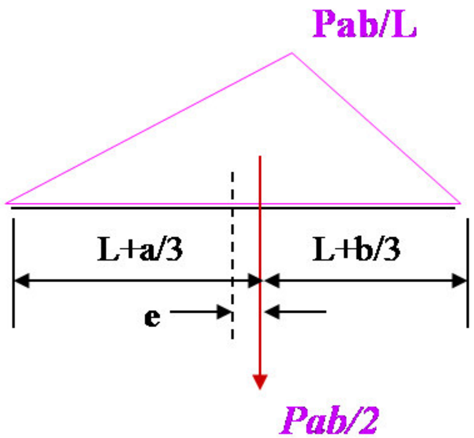
# Example




$A = L$ $\quad I_y = L^3/12 \quad I_x = 0$

$e = 1/3(L/2-a)$




Ms/EI diagram

$P/A = Pab/2L$

$My = (Pab/2)[1/3(L/2-a)]$

$Mx = 0$

At A $\quad Mi = P/A + My = Pab^2/L^2$

At B $\quad Mi = P/A - My = Pba^2/L^2$

$Mab = Ms - Mi = -Pab^2/L^2$

$Mba = Ms - Mi = -Pba^2/L^2$

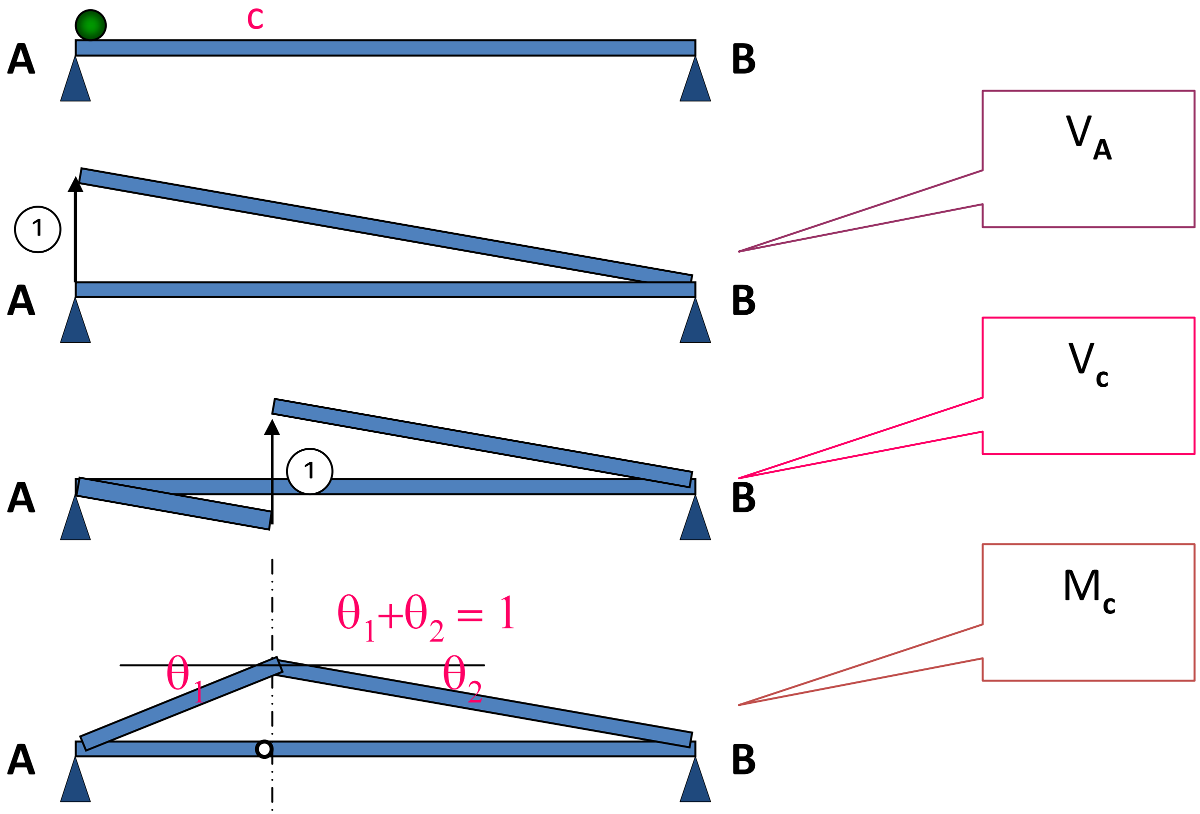
# INFLUENCE LINES For S.D.S

## INFLUENCE LINES FOR S.D.S

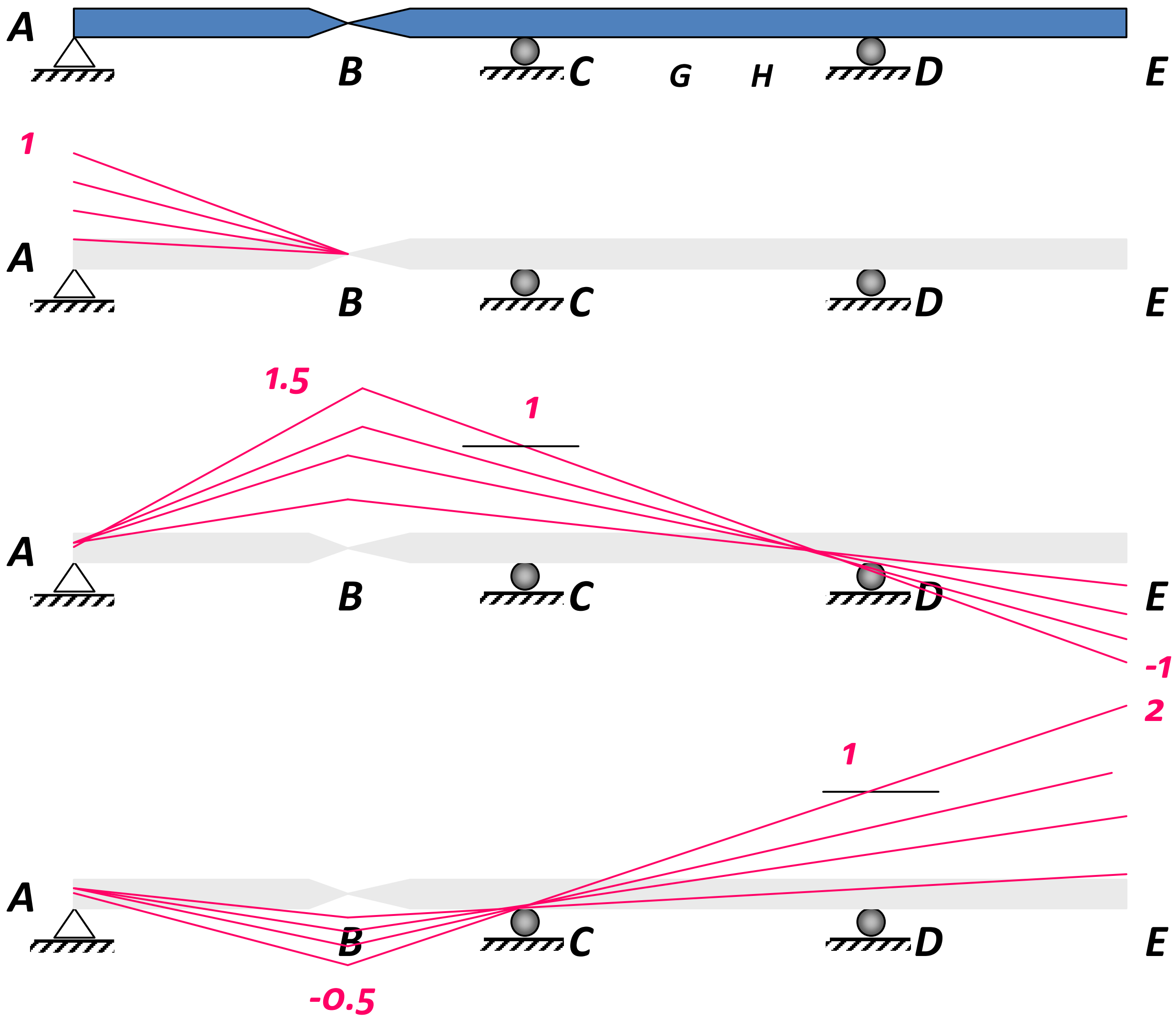
## THE MULLER-BRESLAU PRINCIPLE

Virtual work

“เส้นอิทธิพลของค่าใดๆ(reactions, shear & bending moment)ในโครงสร้างดิเทอร์มิเนท จะมีแนวและรูปทรงเช่นเดียวกันกับแนวและรูปทรงของโครงสร้าง ที่มีการเปลี่ยนแปลงเนื่องจากการยกเลิกความสามารถต่อการต้านทานค่าต่างๆที่จุดพิจารณา ด้วยค่าที่มีขนาดเท่ากับ 1 หน่วย”



# Example



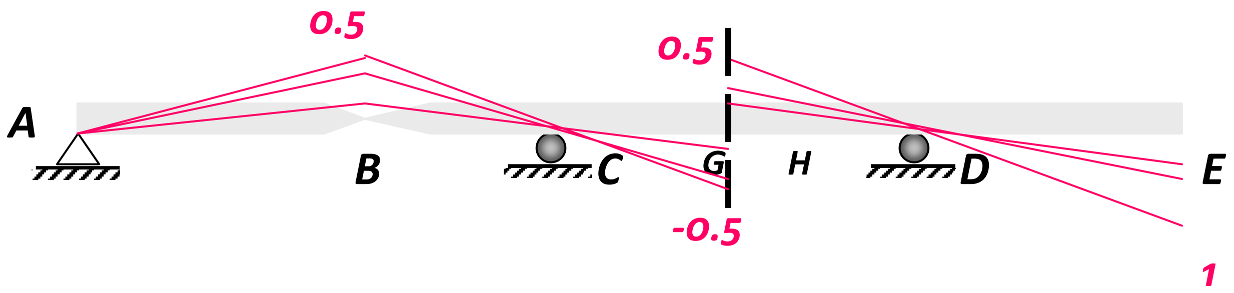

$R_A$

$R_C$

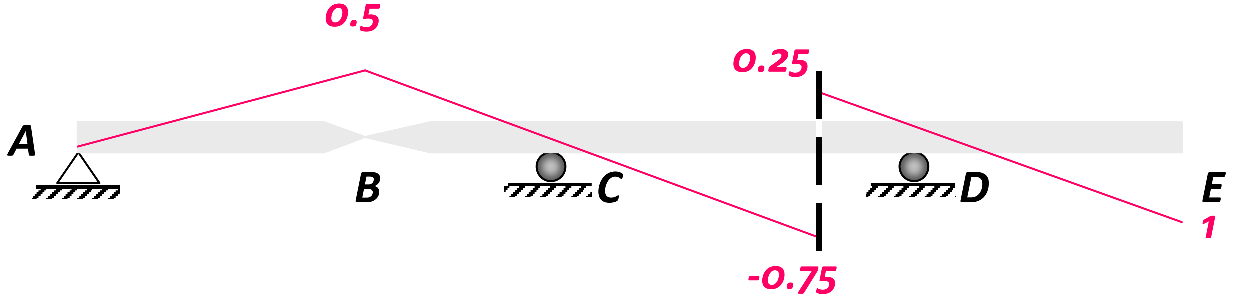
$R_D$

# Example

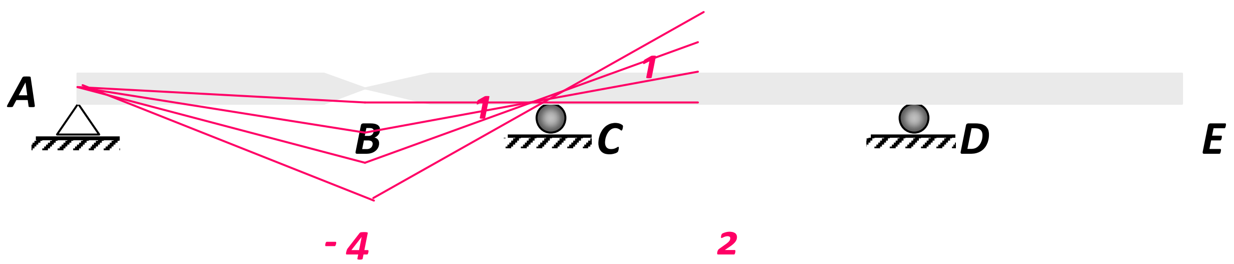
$V_G$



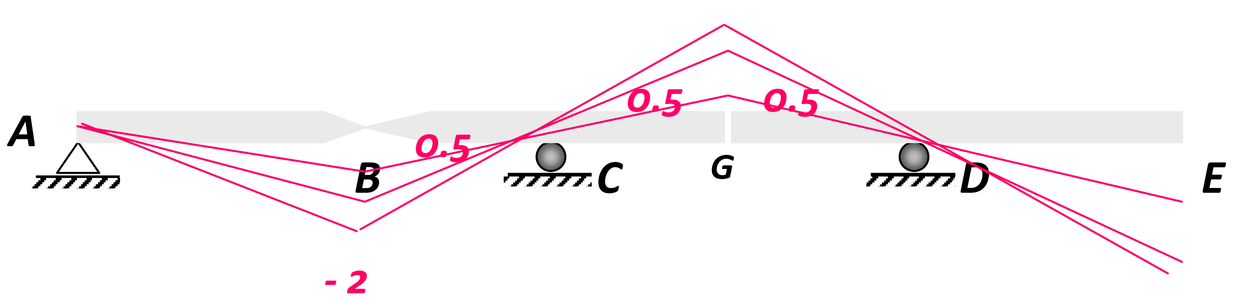

$V_H$



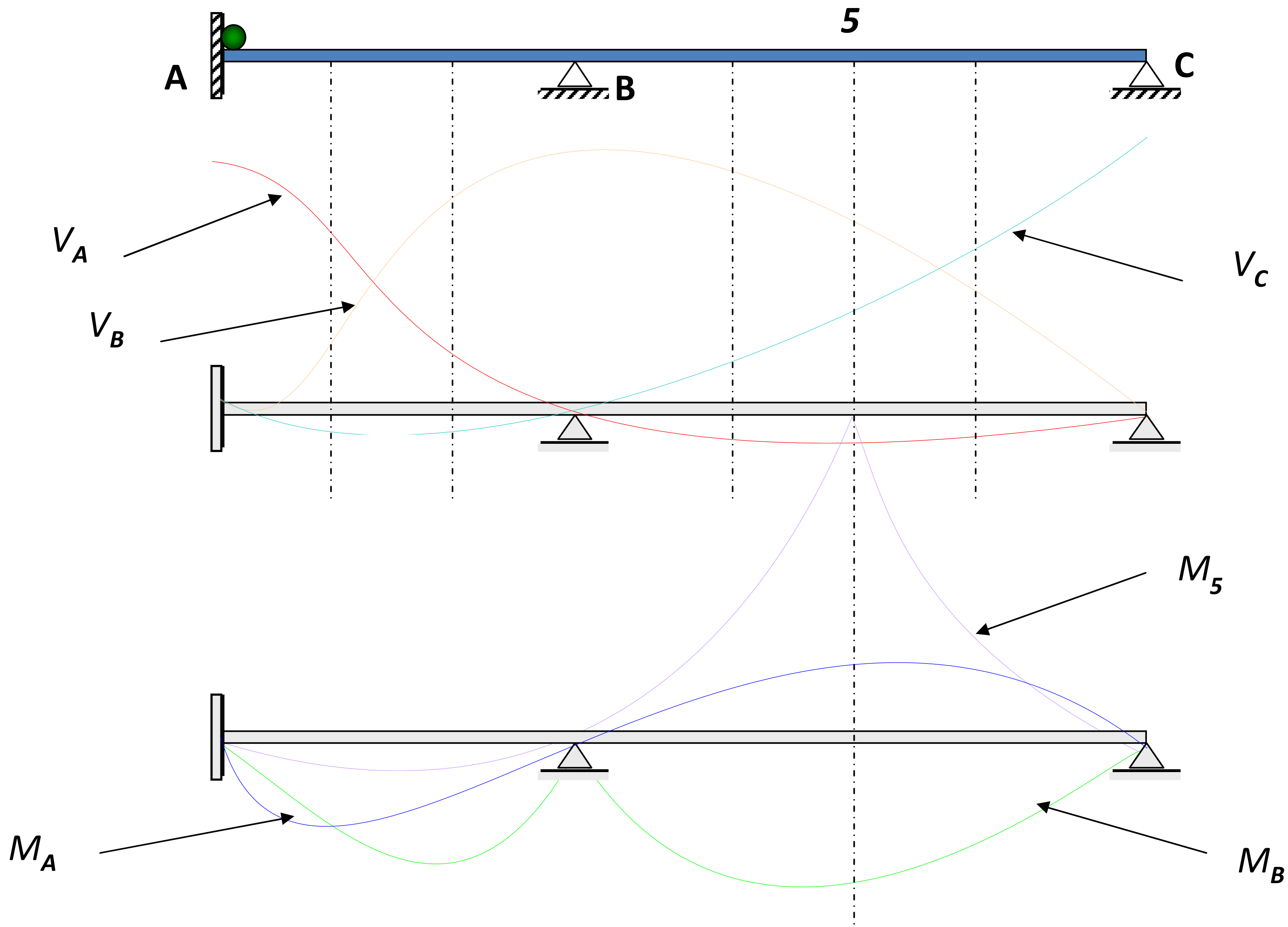

$M_C$




$M_G$



# INFLUENCE LINES FOR S.I.S



# Qualitative Influence Lines and Loading Patterns for an Multi-span Indeterminate Beam

## *Influence lines for reactions*

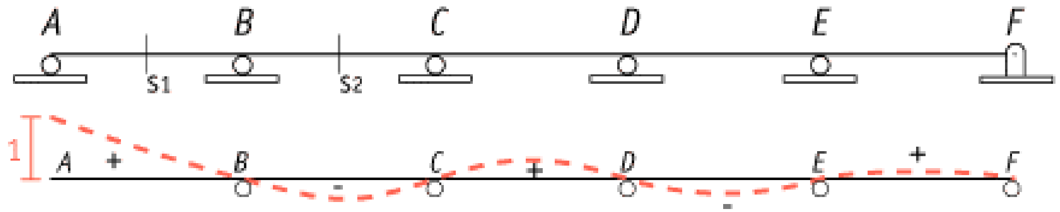



Figure 1 - Structure with support reaction removed, unit deformation applied, and resulting influence line

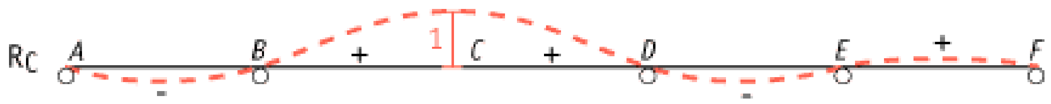



Figure 2 - Structure with support reaction removed, unit deformation applied, and resulting influence line

## *Influence lines for shears*

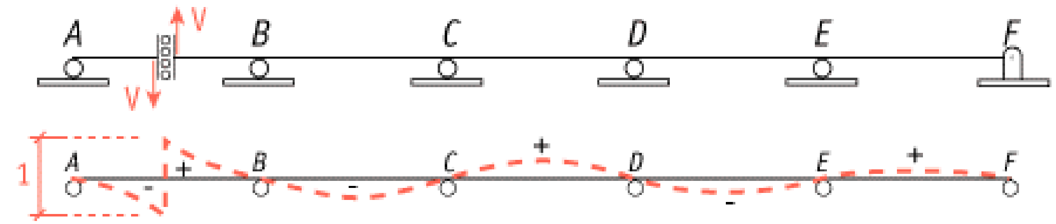



Figure 3 - Structure with shear carrying capacity removed at section S1, deformations applied, and resulting influence line

## *Influence lines for moments*

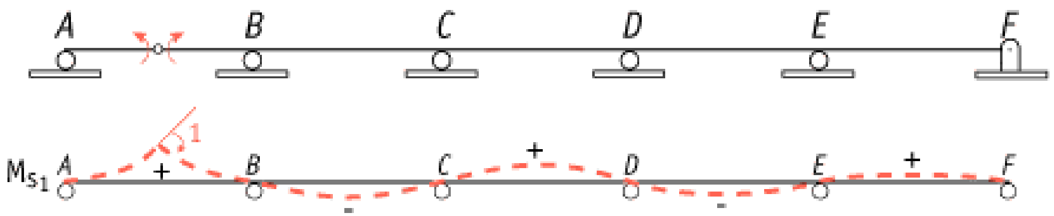



Figure 4 - Structure with moment capacity removed at section S1, unit rotation applied, and resulting influence line

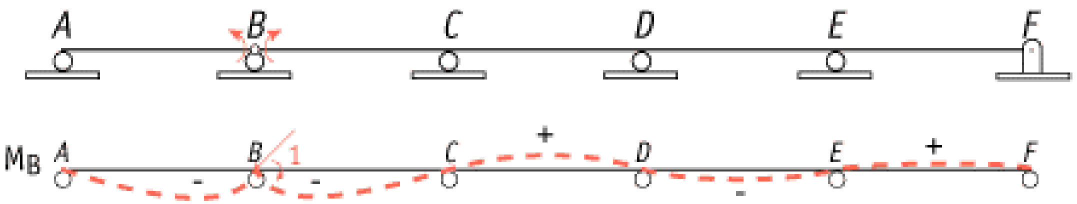



Figure 5 - Structure with moment capacity removed at support B, unit rotation applied, and resulting influence line

***The load cases are generated for the maximum positive and negative values by placing a distributed load on the spans where the algebraic signs of the influence line are the same. i.e., to get a maximum positive value for a function, place a distributed load where the influence line for the function is positive.***

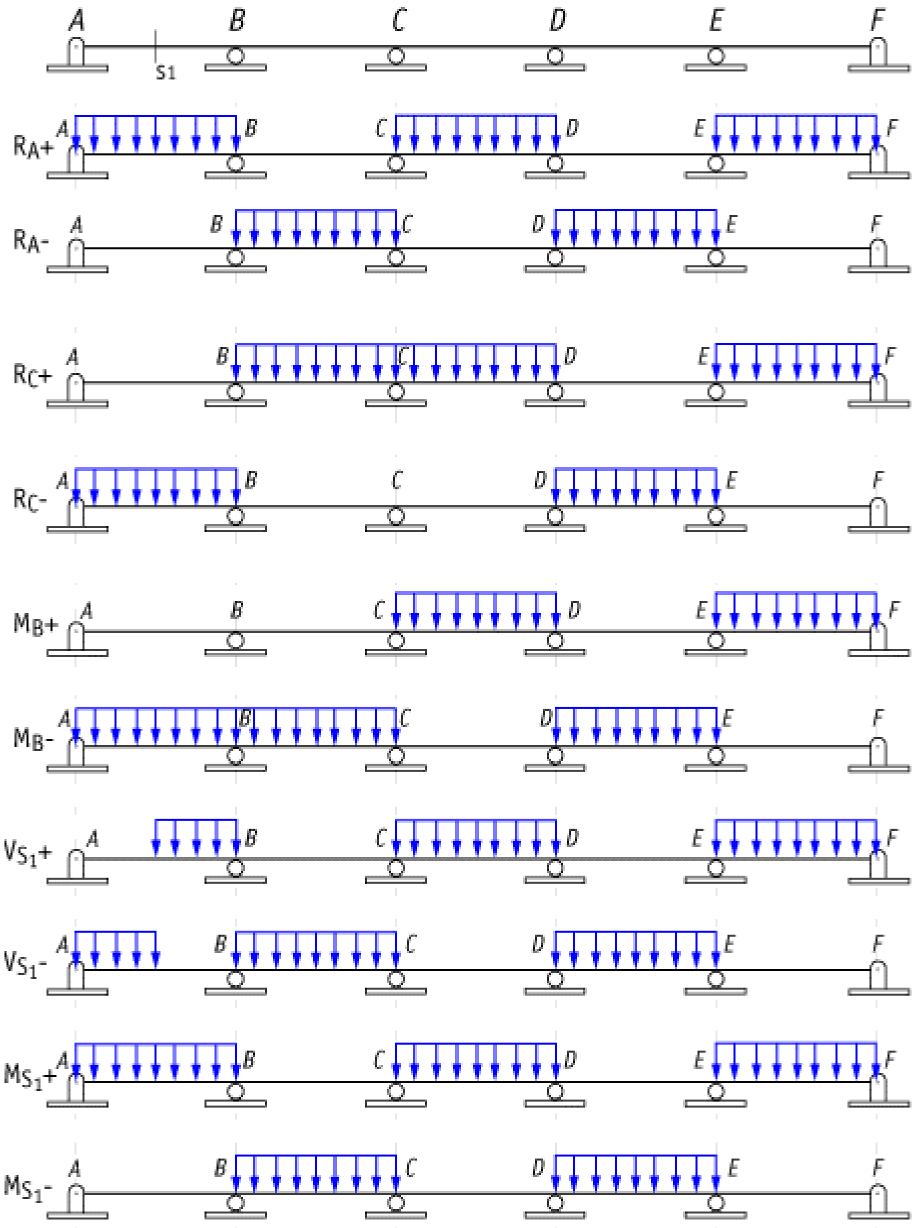



Figure 6 - Multi-span structure

Figure 7 - Maximum positive reaction at support A

Figure 8 - Maximum negative reaction at support A

Figure 9 - Maximum positive reaction at support C

Figure 10 - Maximum negative reaction at support C

Figure 11 - Maximum positive moment at support B

Figure 12 - Maximum negative moment at support B

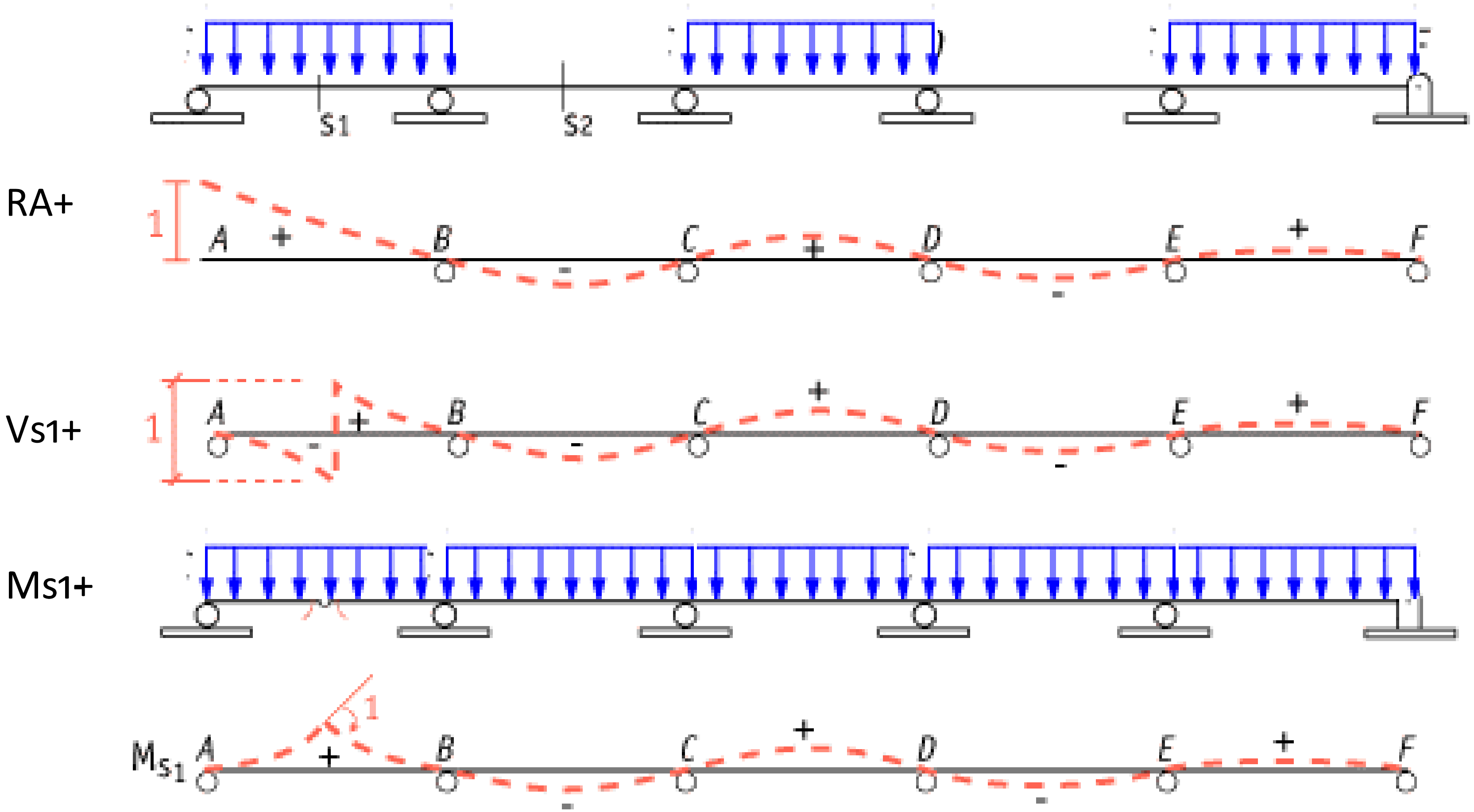
Figure 13 - Maximum positive shear at s

Figure 14 - Maximum negative shear at s

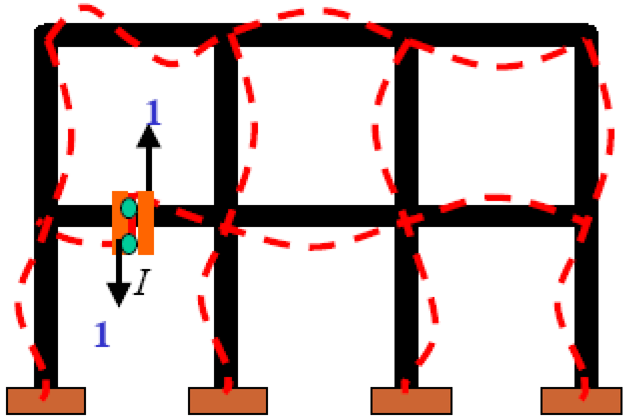
Figure 16 - Maximum positive moment at s

Figure 17 - Maximum negative moment at s

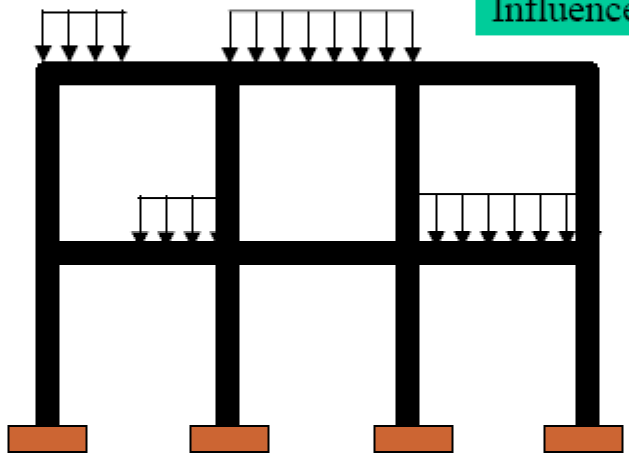
# EXAMPLE



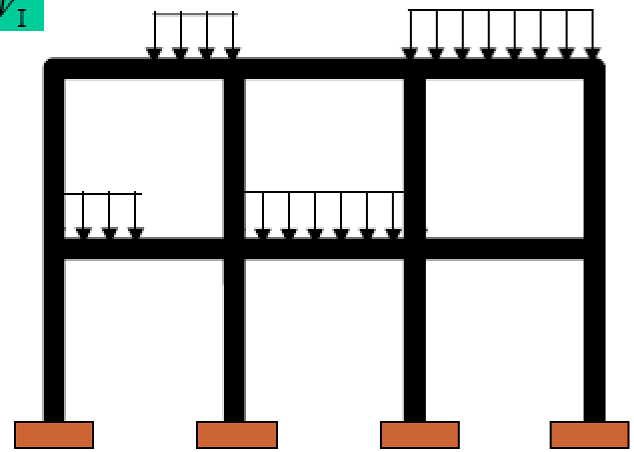
## Qualitative Influence Lines for Frames




Influence Line of $V_I$

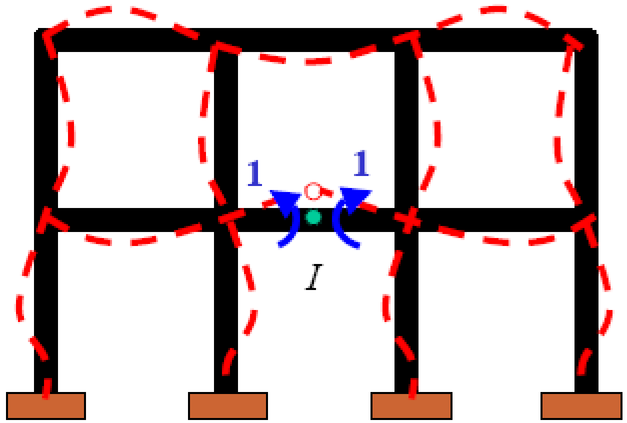


Maximum **positive** shear

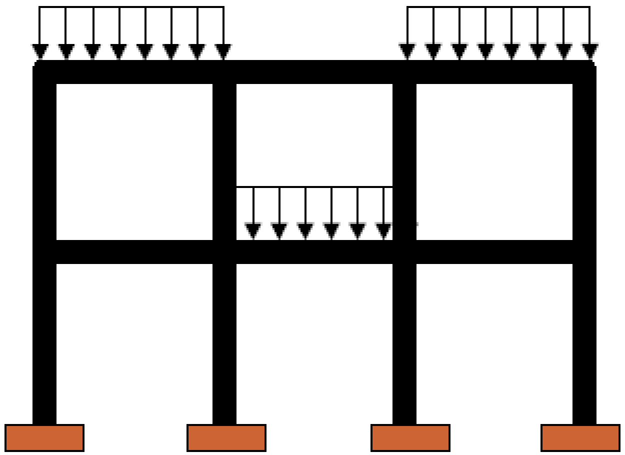


Maximum **negative** shear

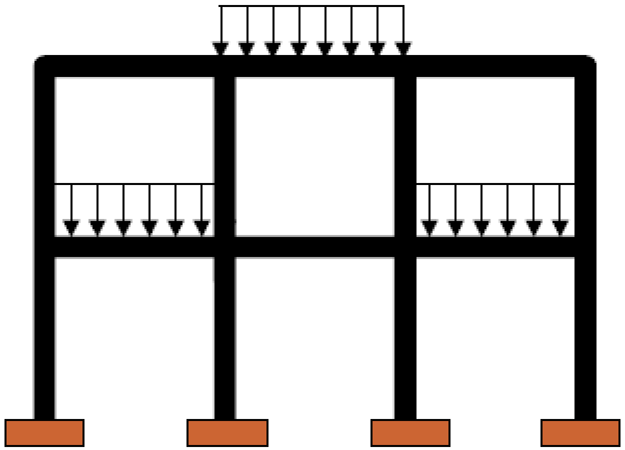
## Qualitative Influence Lines for Frames




Influence Line of $M_I$

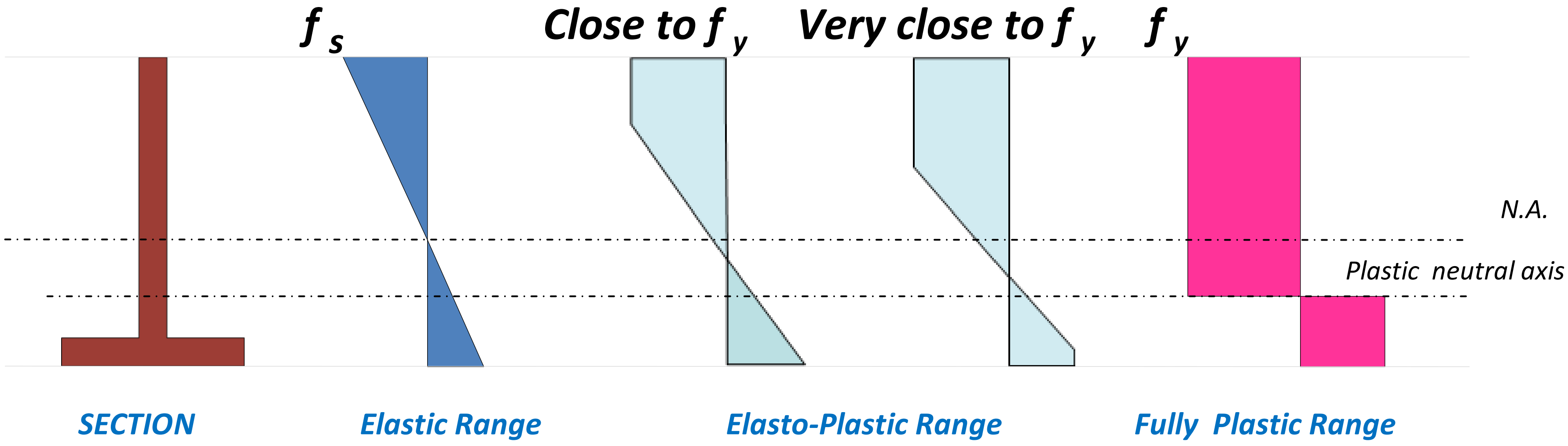


Maximum **positive** moment



Maximum **negative** moment

# PLASTIC METHOD




$f = Mc/I = M/S$

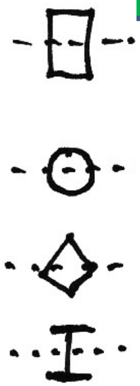
$f_y = M_p / Z_p$

$Z f_y / S f_y = M_p / M = Z / S$ — F = Shape Factor

$w_p / w$ — Load Factor

*Elastic Moment* $M = ½\, b\ h/2\ \ 2/3\, h\, f_y = 1/6\ bh^2\ f_y$

*Plastic Moment* $M_p = b\ h/2\ \ ½\ h\, f_y = 1/4\ bh^2\ f_y$



Rectangle: *f =1.5,*

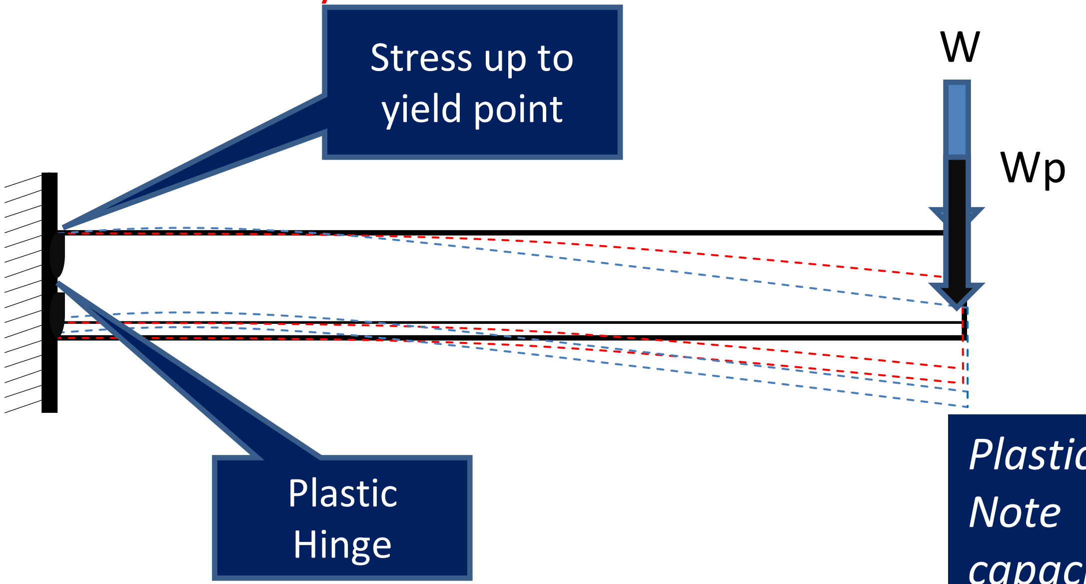
Circle: *f =1.698;*

Diamond: *f = 2.0;*

Steel I-beam: *f is between 1.12 and 1.15.*

# COLLAPSE MECHANISM

A structure is deemed to have reached the limit of its load carrying capacity When it forms sufficient hinges to convert it into a mechanism with consequent Collapse. This is normally one hinge more than the number of degree-of-indeterminacy.



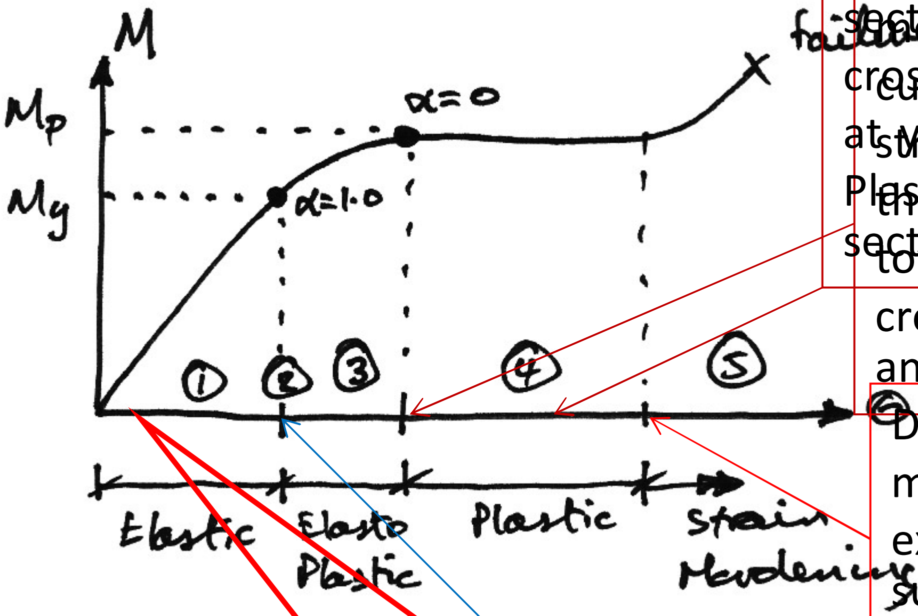

*Plastic Hinge*

*Note that once the plastic moment capacity is reached, the section can rotate freely –that is, it behaves like a hinge, except with moment of $M_p$ at the hinge. This is termed a plastic hinge, and is the basis for plastic analysis. AT the plastic hinge*

*stresses remain constant, but strains and hence rotations can increase.*

$M_R$ = Elastic Moment of Resistance $\Rightarrow$ W

$M_P$ = Plastic Moment of Resistance $\Rightarrow$ $W_p$




The applied moment causes stresses over the cross-section that are all less than the yield stress of the material

The applied moment is just sufficient that the yield stress of the material is reached at the outermost fibres of the cross-section. All other stresses in the cross section are less than the yield stress. This is limit of applicability of an elastic analysis and of elastic design. Since all fibres are elastic,.

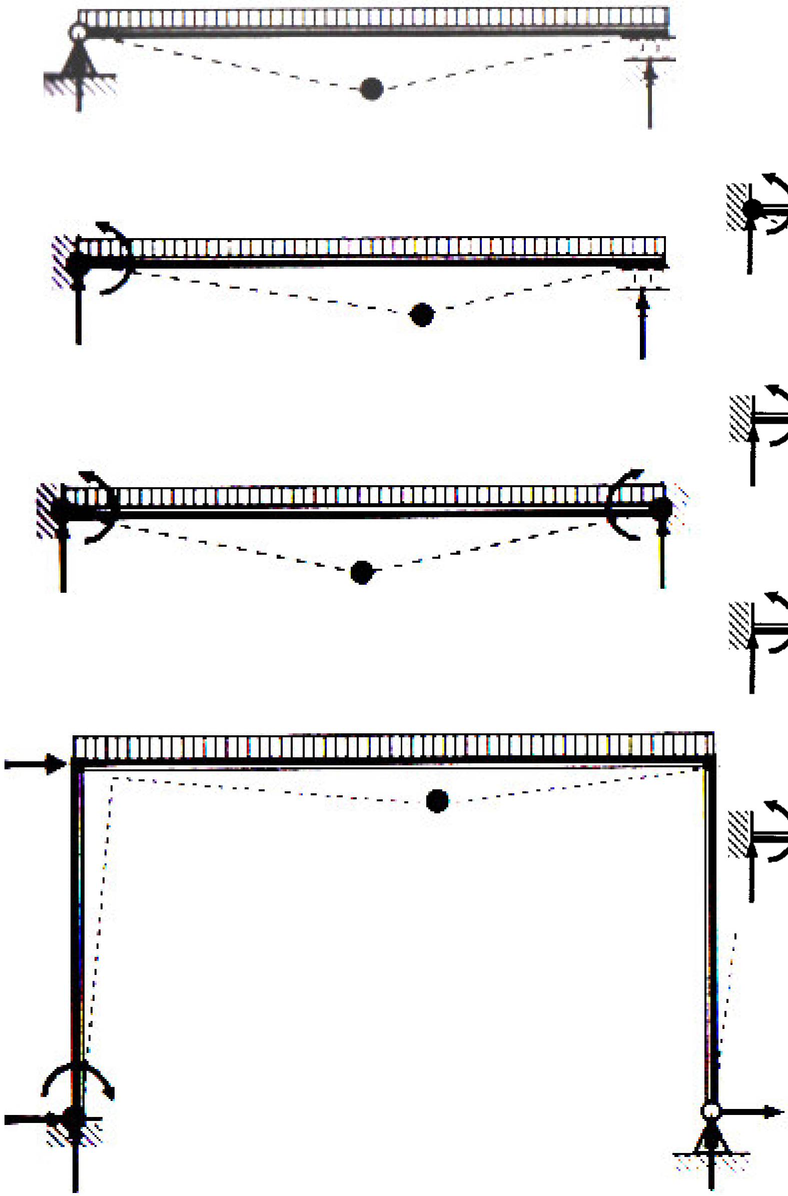
The moment applied to the cross section has been increased beyond the yield moment. Since by the idealised stress-strain curve the material cannot sustain a stress greater than yield stress, the fibres at the yield stress have progressed inwards towards the centre of the beam. Thus over the cross section there is an elastic core and a plastic region.

The applied moment to the cross section is such that all fibres in the cross section are at yield stress. This is termed the Plastic Moment Capacity of the section.

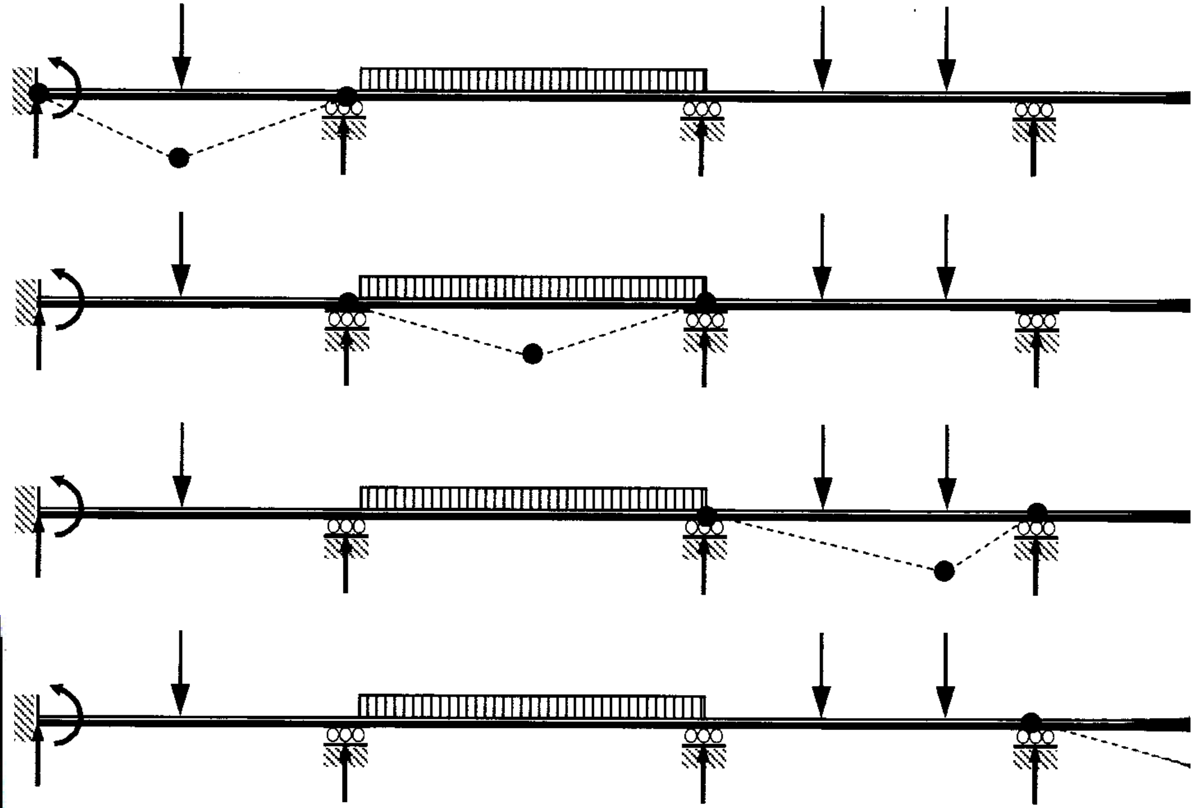
Due to strain hardening of the material, a small amount of extra moment can be sustained.

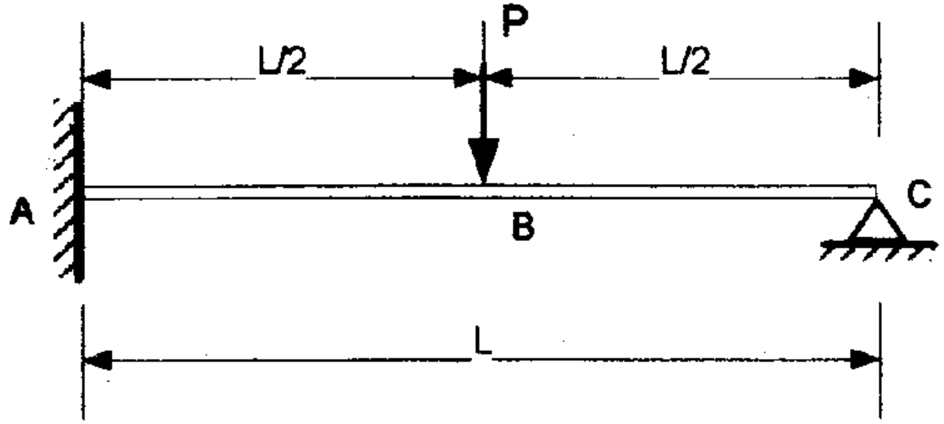
Minimum number of hinges required $(I_D + 1) = 1$

## Full Plastic

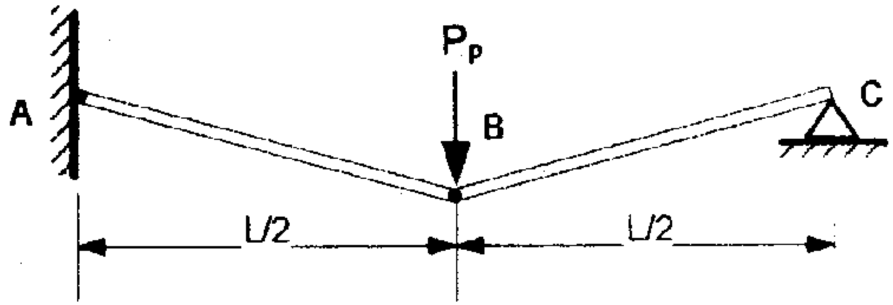


## Partial Plastic

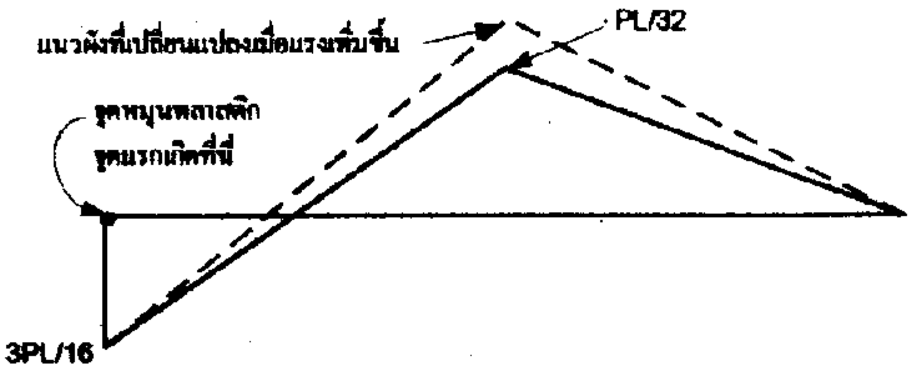




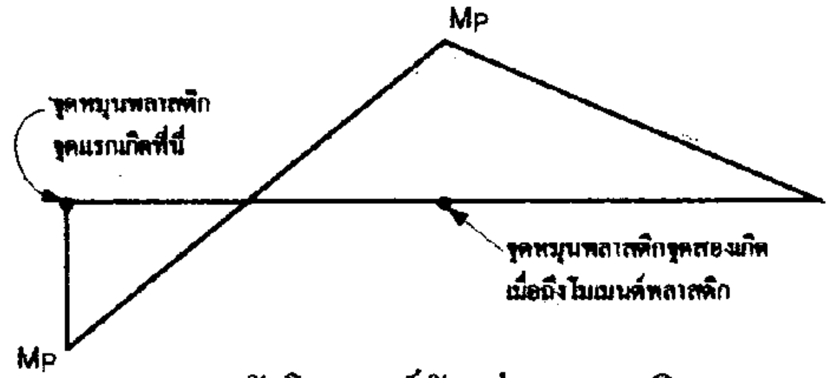

(ก) คานตัวอย่าง




(ง) กลไกการวิบัติของคาน

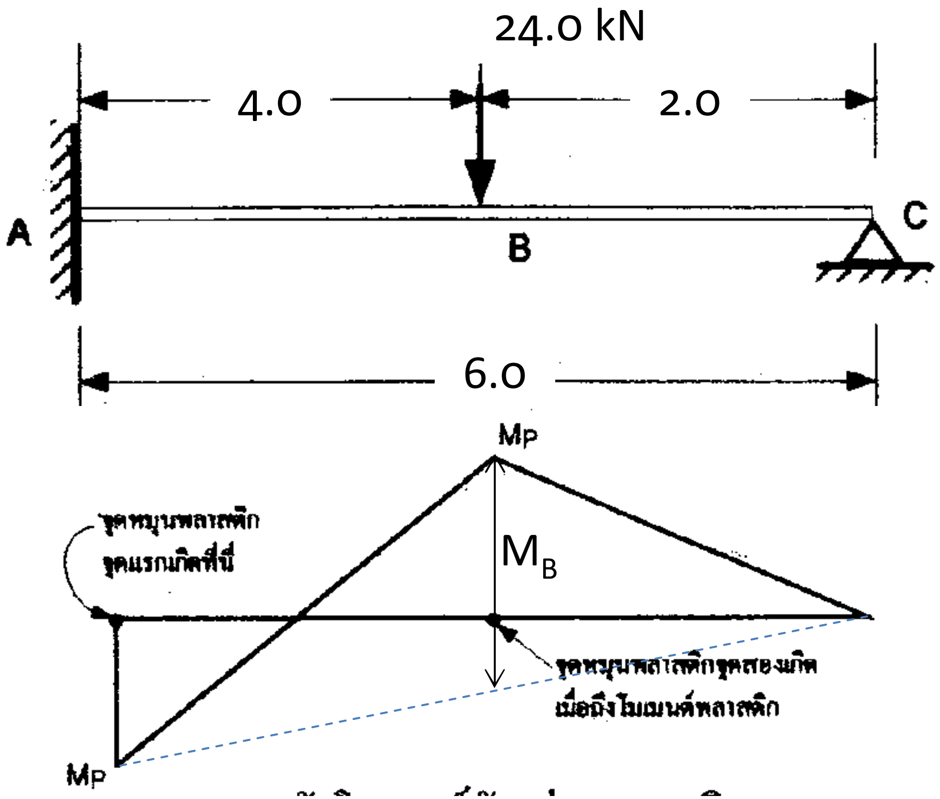



(ข) ผังโมเมนต์ดัด(ช่วงอีลาสติก)

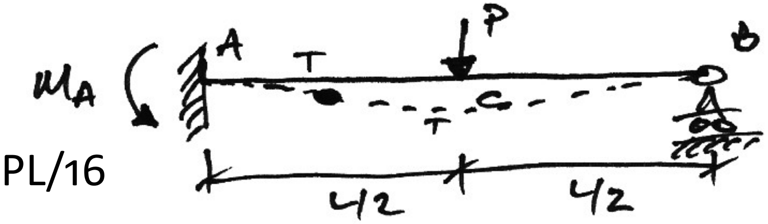



(ค) ผังโมเมนต์ดัด(ช่วงพลาสติก)

Example : Determine Mp if 24.0 kN is collapse load.

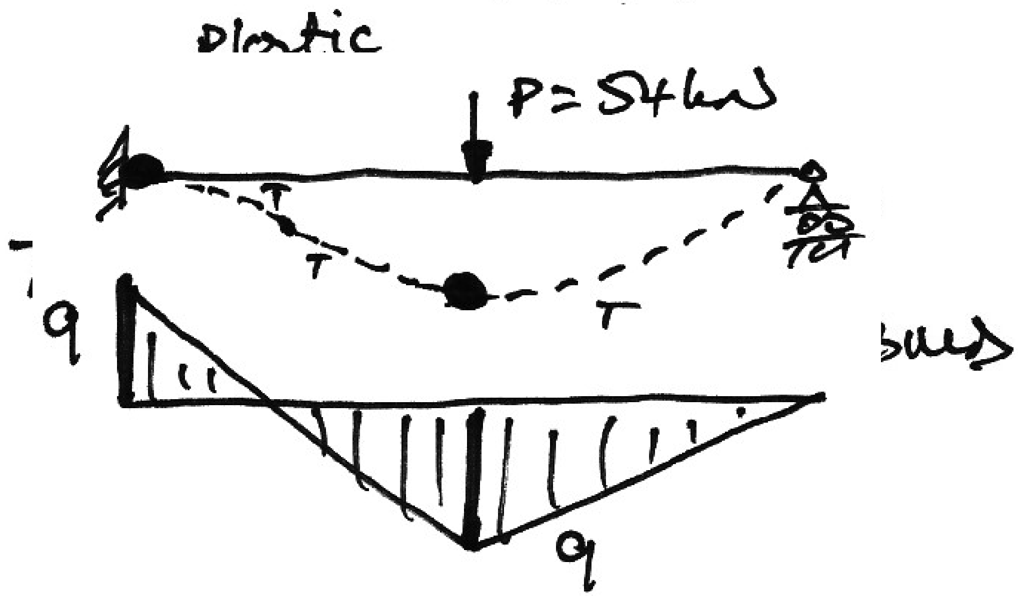



1. The collapse load = 24 kN
2. The number of hinges required to induce collapse = $(I_D + 1) = 2$
3. $V_A = 8.0$ kN , $V_C = 16.0$ kN
4. $M_A = M_B = Mp$
5. $(MB + 0.333MA) = (Mp + 0.333Mp) = 1.333Mp = 32$
6. So $Mp = 32/1.333 = 24$ kNm




$M_A = 3PL/16$

$M_C = 5PL/32$



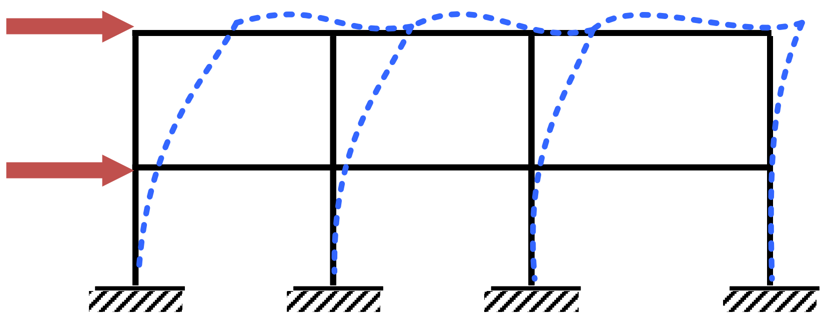
# Approximate Method

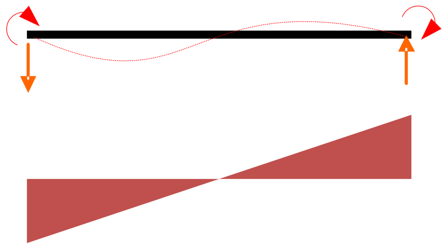
| | | | | |
|---|---|---|---|---|
| Vertical Loads | ➡ | *Gravity Method* | ➡ | *Concept of Inflection points* |
| Horizontal Loads | ➡ | *Portal Method* | ➡ | *Concept of Inflection points At mid span or mid height* |
| | | *Cantilever Method* | ➡ | *Concept of stresses in beam* |

## Assumption

1. Sideway occurred very fast from lower to upper storey. Every connection joints will rotate clockwise.
2. End moment of columns will counter clockwise, because of horizontal shear forces must be right to left.

   Only one Inflection point occurred in column.
3. End moment of beams will clockwise, only one inflection point occurred.

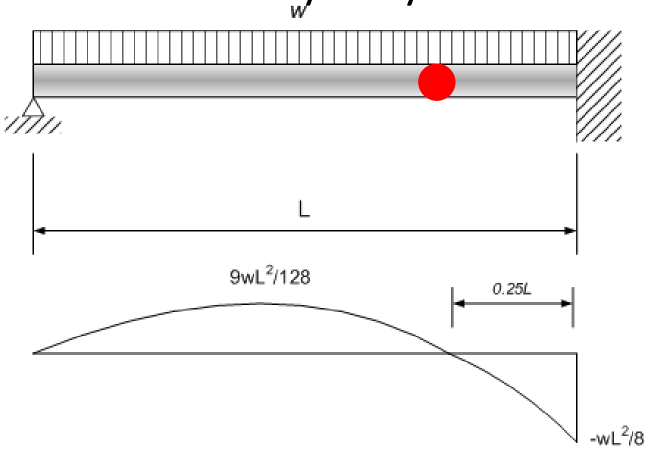


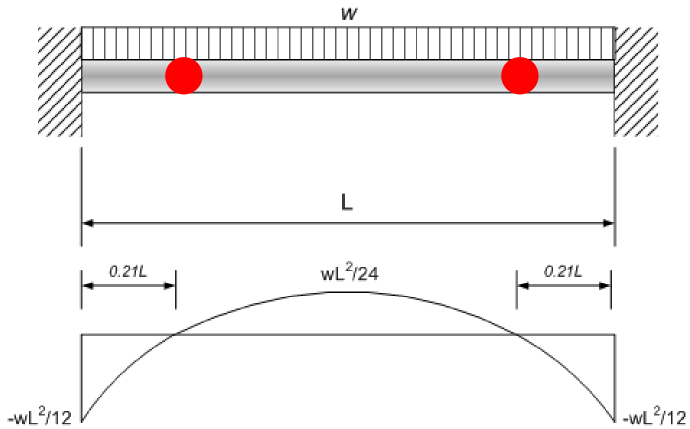


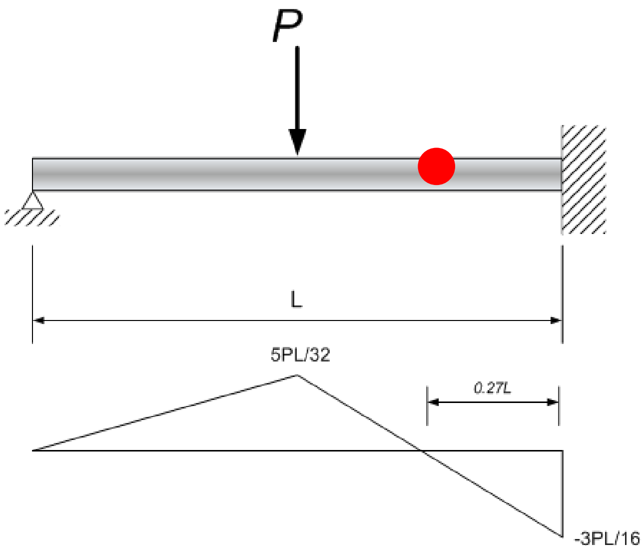
# GRAVITY METHOD

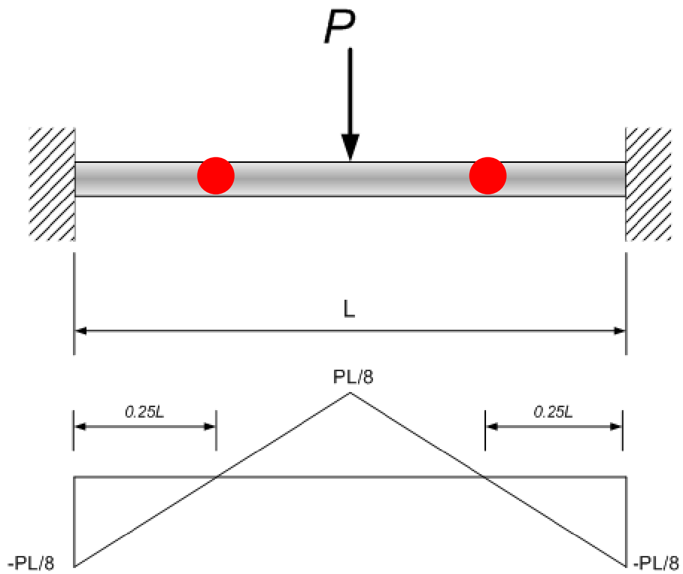
*(For Continuous Beam)*

By adding hinges equal in number to
The degree of indeterminacy, and
The structure convert to primary structure
Then we can analyze by statics.

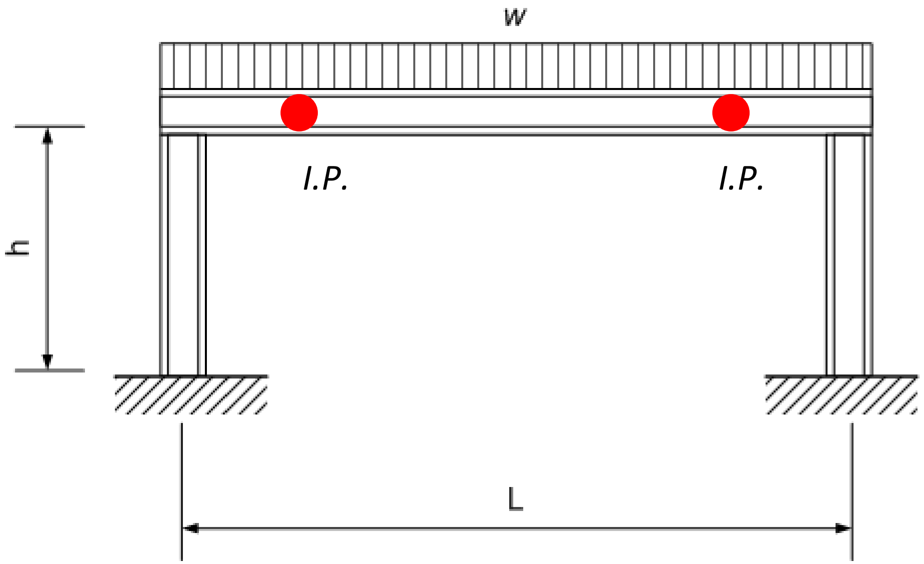


# GRAVITY METHOD *(For Rigid Frame)*



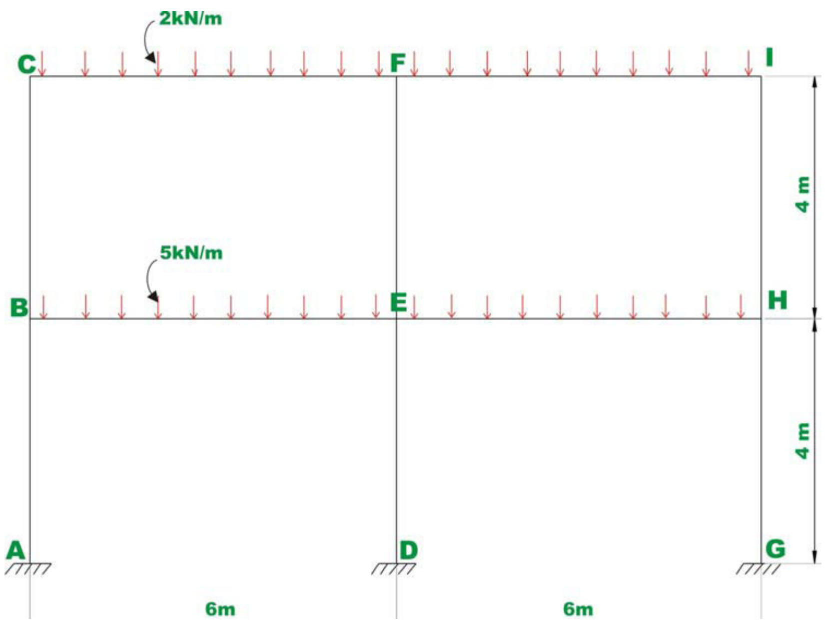

Column stiffness = $I_c / h$ (C)

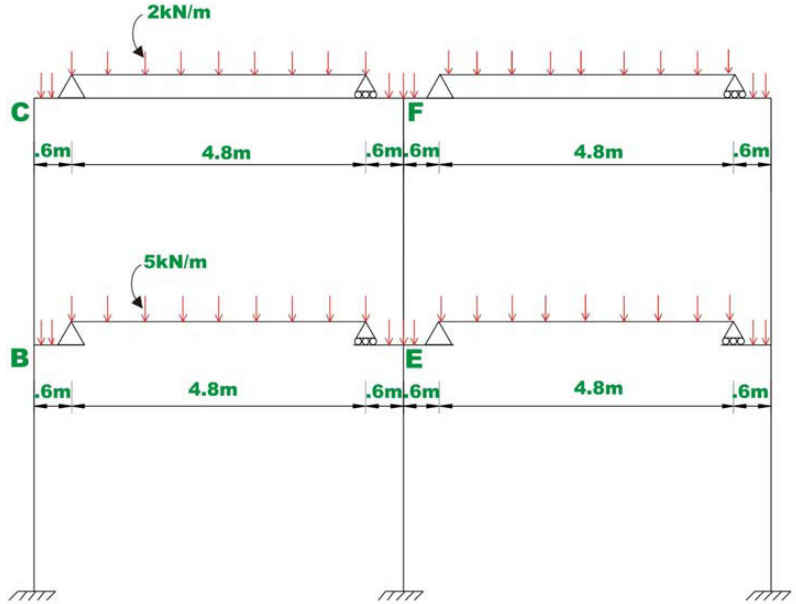
Girder stiffness = $I_g / L$ (G)

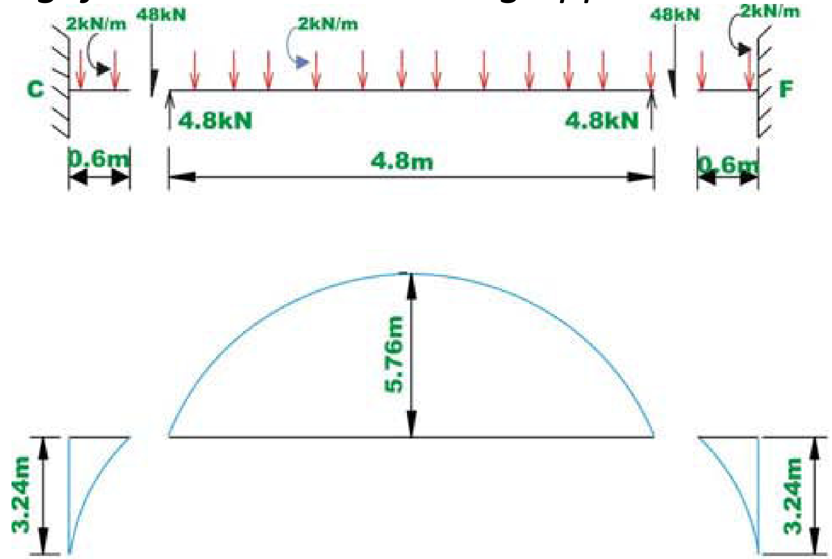
*If (c) > (G)* FEM of Girder = 70-85% of the end moments in A fixed end beam of the same span.(I.P = 0.2L)

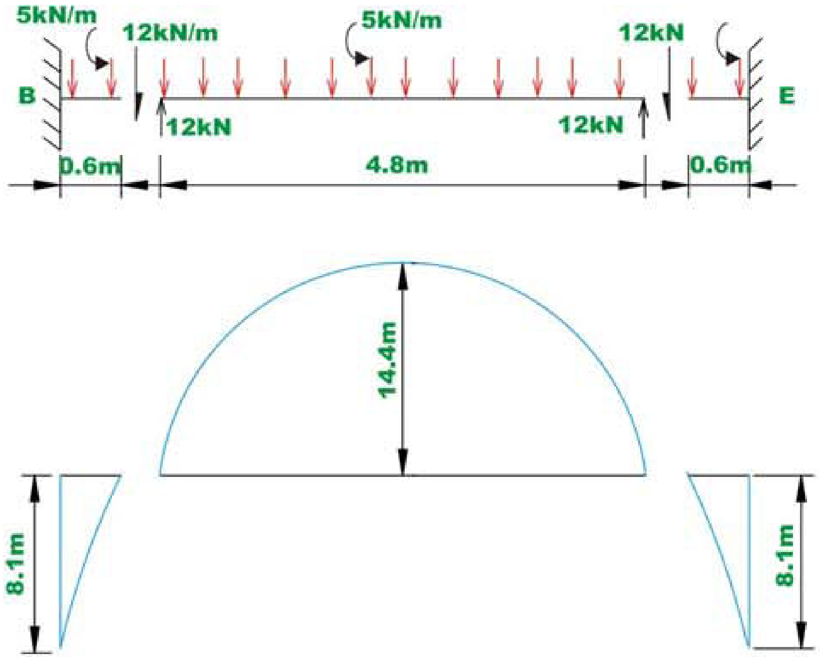
*If (G) > (C)* FEM of Girder = 15-25% of the end moments in A fixed end beam of the same span.(I.P.=0.1L to 0.15L)

***Example*** *Analyze the building frame shown in Fig. for vertical loads using approximate methods.*



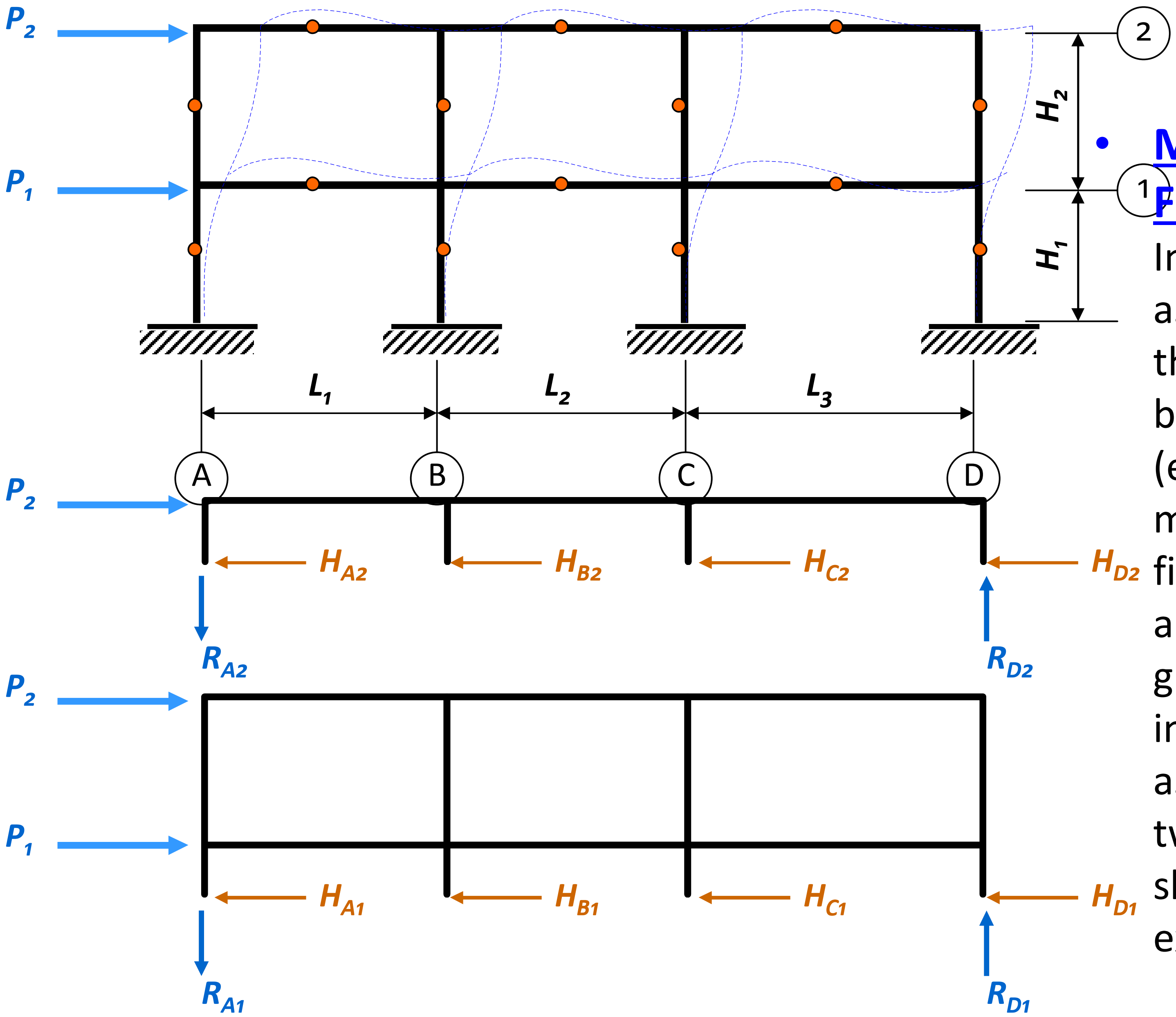



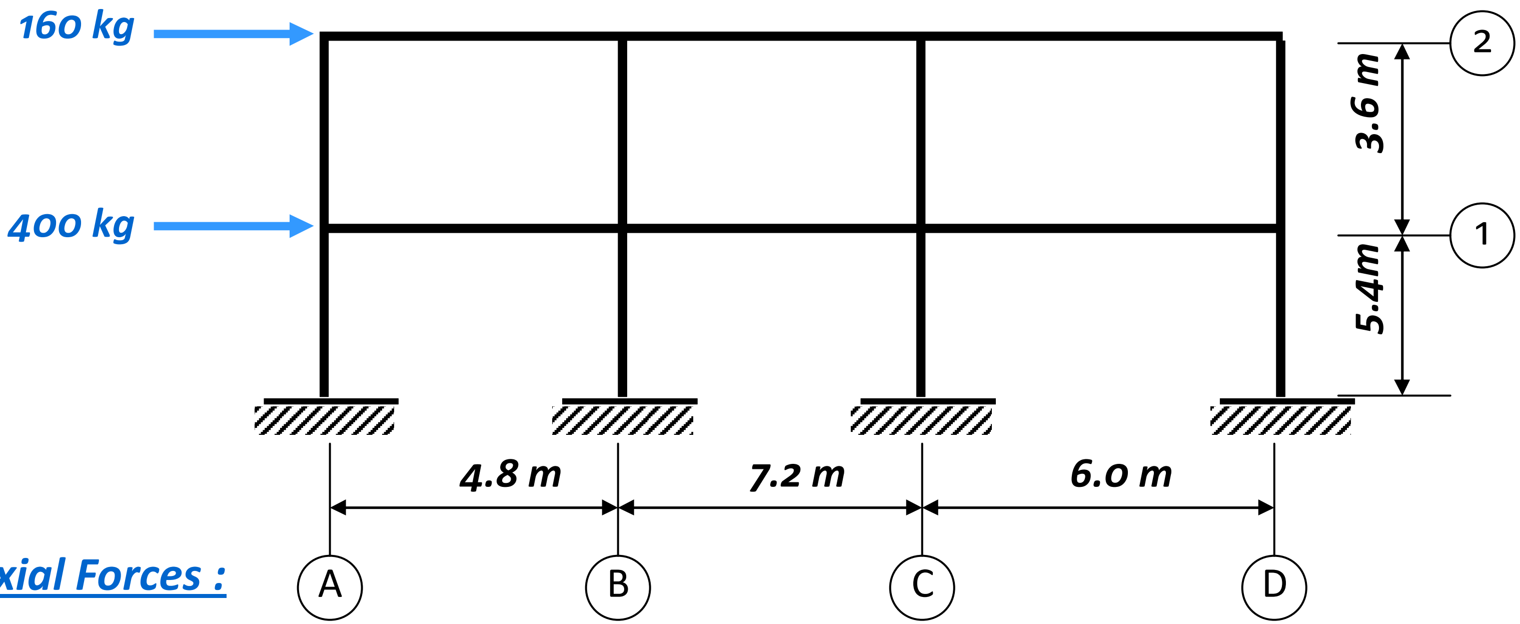

**Bending moment diagrams**

# PORTAL FRAME METHOD

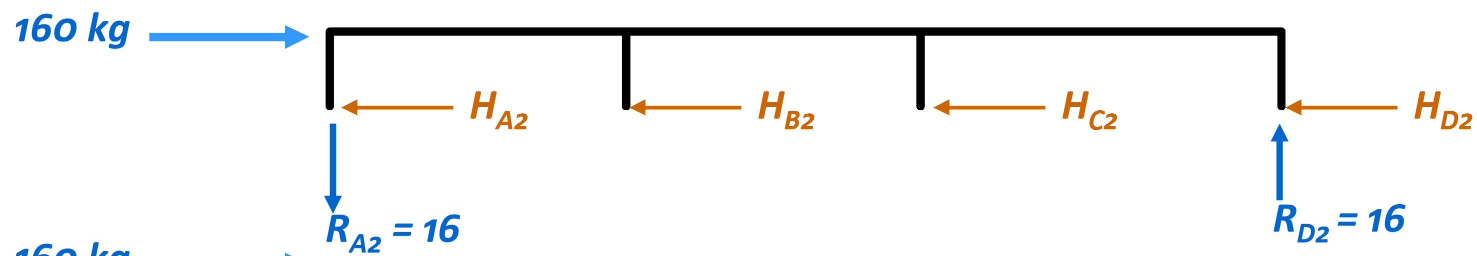


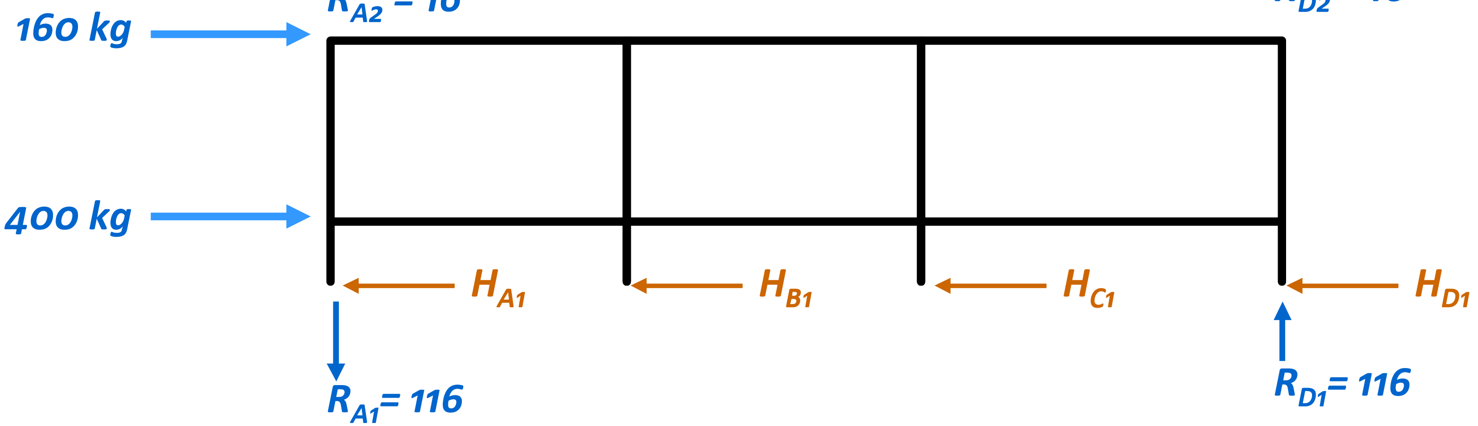

- **Method I - Portal Frame Method:** Inflection points are assumed to occur at the middle points of beams and columns (earlier assumptions made for partial fixity at base are also valid) - At any given floor level, interior columns are assumed to carry twice the horizontal shear carried by the exterior columns.

## Example

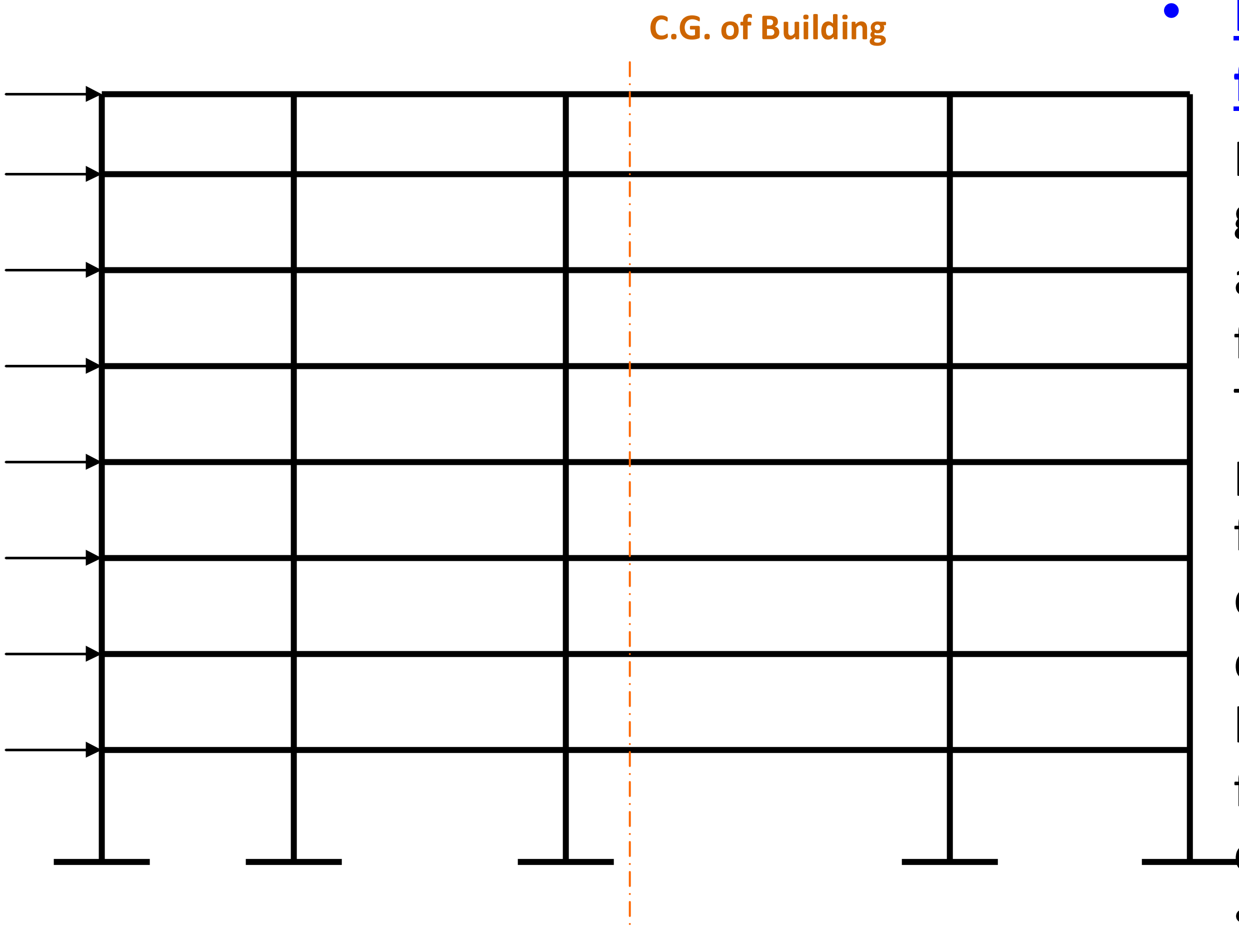



## Axial Forces :



# CANTILEVER METHOD




*Assumption that a building frame behaves as a cantilever beam*

- **Method II - Cantilever frame method:** Hinges are placed at the center of each girder and column (earlier assumptions made for partial fixity at base are also valid) - The axial stress in a column is proportional to its distance from the cancroids of the cross-sectional areas of the columns at a given floor level; since stress equals force per area, in the case of columns having equal cross-sectional areas, the force in a column is also proportional to its distance from the cancroids of the column areas.

***These the following steps are for analyzing a frame by <u>the cantilever method</u> :***

***1.Cut free bodies of each story together with the upper and lower halves of the attached columns. The free bodies are cut by passing sections through the middle of the columns(midway between floor). Since the sections pass through the point of inflection, only axial and shear forces act on each column at that point.***

***2. Evaluate the axial force in each column at the point of inflection in a given story by equating the internal moments produced by the column forces to the moment produced by all lateral loads above the section.***

***3. Evaluate the shears in the girders by considering vertical equilibrium of joints. The shears in the girders equals the difference in axial forces in the columns. Start at an exterior joint and proceed laterally across the frame.***

***4. Compute the moments in the girders.*** $M = V(L/2)$

***5. Evaluate the column moments by considering equilibrium of joints. Start with the exterior joints of the top floor and proceed downward.***

***6. Establish the shears in the columns by dividing the sum of the column moments by the length of the column.***

***7. Apply the column shears to the joints and compute the axial forces in the girders by considering equilibrium of forces in the x direction.***

# Introduction to the General Stiffness Method

Stiffness Method (Displacement Method of Analysis)

The displacement method can be applied to statically determinate or indeterminate structures, but is more useful in the latter, particularly when the degree of statically indeterminacy is high.

In this method, one must first determine the degree of kinematic indeterminacy. A coordinate system is then established to identify the location and direction of joint displacements. Restraining forces equal in number to the degree of kinematic indeterminacy are introduced at the co-ordinates to prevent the displacement of the joints. The restraining forces are finally determined as a sum of the fixed end forces for the members meeting at a joint. (For most practical cases, the fixed-end force can be calculated with the aid of standard tables)

*Stiffness Matrix [S]*

*{D}= [S] {- F} -1*

*The elements of the vector {D} are the unknown displacements.*

*The elements of the matrix [S] are forces corresponding to unit values of displacements.*

*The column vector {F} depends on the loading on the structure*

In general cases, the number of restraints introduced in the structure is *n, the order of the* matrices {D}, [S] and {F} is *n x 1, n x n and n x 1 respectively.*

The general steps followed in an analysis using the stiffness method are as follows:

o establish a relationship between the element forces and displacements (e.g. between moments and rotations, forces and deflections)

o Reassemble the elements to form original structure & apply compatibility to the joints.

o Apply equilibrium on the assembled structure at each joint.

# Introduction to the General Stiffness Method

Flexibility Method (Force Method of Analysis)

In this method, the degree of statically indeterminacy is initially determined. Thereafter, a number of releases equal to the degree of statically indeterminacy is introduced, each release being made by the removal of an external or internal force. The magnitude of inconsistencies introduced by the releases is the determined. Next, the displacements in the released structure due to unit values of the redundants are determined. This allows the values of the redundant forces necessary to eliminate the inconsistencies in the displacements to be determined. Hence, the forces on the original indeterminate structure are calculated as the sum of the correction forces (redundants) and forces on the released structure.

*Flexibility Matrix [f]*

*[f ]{F}= {D -D}*

*D* represents inconsistencies in deformation while {F} represents the redundants.

*D* elements represent prescribed displacements at their respective coordinates.

The column vector {D - D} thus depends on the external loading.

The elements of the matrix *[f] are displacements due to the unit values of the redundants.*

Therefore *[f] depends on the properties of the structure, and represents the flexibility of* the released structure. For this reason, *[f] is called the flexibility matrix and it's elements* are called flexibility coefficients.

The general steps followed in an analysis using the flexibility method are as follows:

o The structure is rendered indeterminate by the insertion of suitable releases,and is now called the primary structure (e.g. insert three releases for a degree of redundancy of three)

o By inserting a release, a condition of compatibility at that point is abandoned. Since the primary structure is now statically determinate, a solution is carried out and the member forces are calculated by applying equilibrium conditions only.

o Release forces are introduced in the structure so as to restore conditions of compatibility at the releases. A complementary solution of the secondary structure is now carried out. Here, the displacements at the releases due to the release forces only are calculated.

o Next, the solutions of the primary structure and the complementary solution are combined to give the total displacement at the releases due to both the applied loads and the release forces. Finally, the member forces in the original structure may be obtained by the superposition effects from the particular and complementary solutions.

Choice of Force or Displacement Method

In some structures, the formation of one of the matrices – stiffness or flexibility – may be easier than the formation of the other. This situation arises from the following general considerations. In the force method, the choice of the released structure may affect the amount of calculation. For example, in the analysis of a continuous beam, the introduction of hinges above indeterminate supports produces a released structure consisting of a series of simple beams. In other structures, it may not be possible to find a released structure for which the redundants have a local effect only. In the displacement method, generally all joint displacements are prevented regardless of the choice of the unknown displacement. A displacement of a joint affects only the members meeting at the given joint. These properties generally make the displacement method easy to formulate, and it is for this reason that the displacement method is more suitable for computer programming.

# Matrix Analysis by the Direct Stiffness Method

## PLANE TRUSS ANALYSIS


*From Elastic Theory:*

*If we hold* $\delta_1 = 0$ $\quad f_2 = -f_1 = \frac{AE}{L}\delta_2$ $\qquad$ *If we hold* $\delta_2 = 0$ $\quad f_1 = -f_2 = \frac{AE}{L}\delta_1$

*If both* $\delta_1$ & $\delta_2$ *are induced, By superposition :*

$f_1 = \frac{AE}{L}(\delta_1 - \delta_2)$ $\qquad$ $f_2 = \frac{AE}{L}(\delta_2 - \delta_1)$

Force- displacement relation

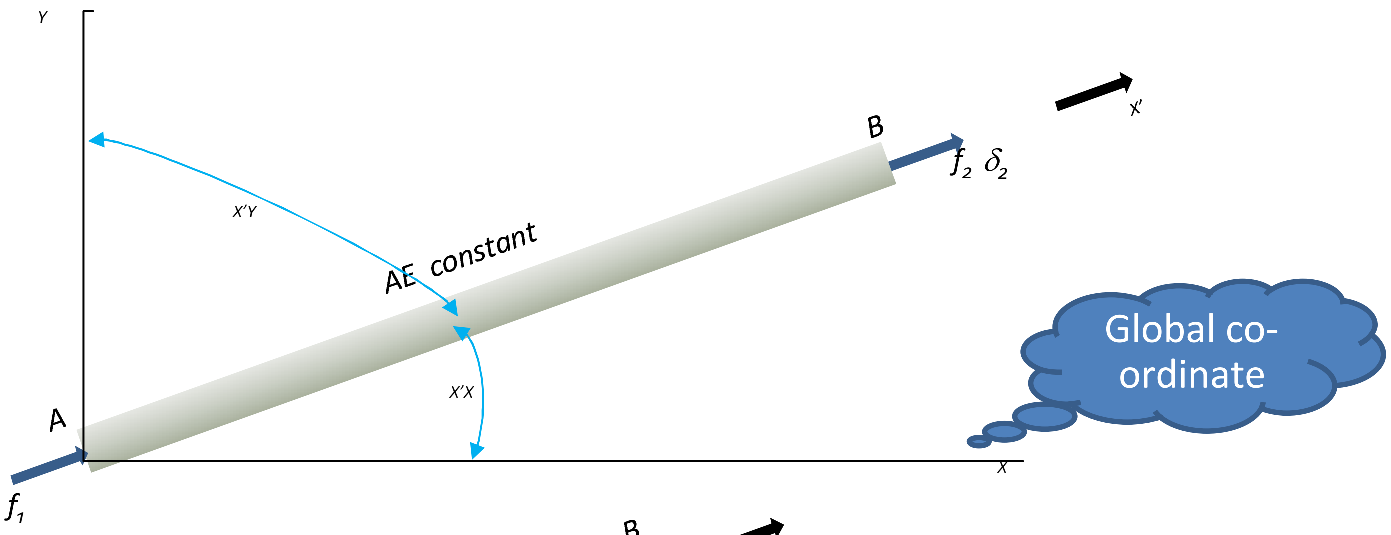
*Writing in Matrix form:*

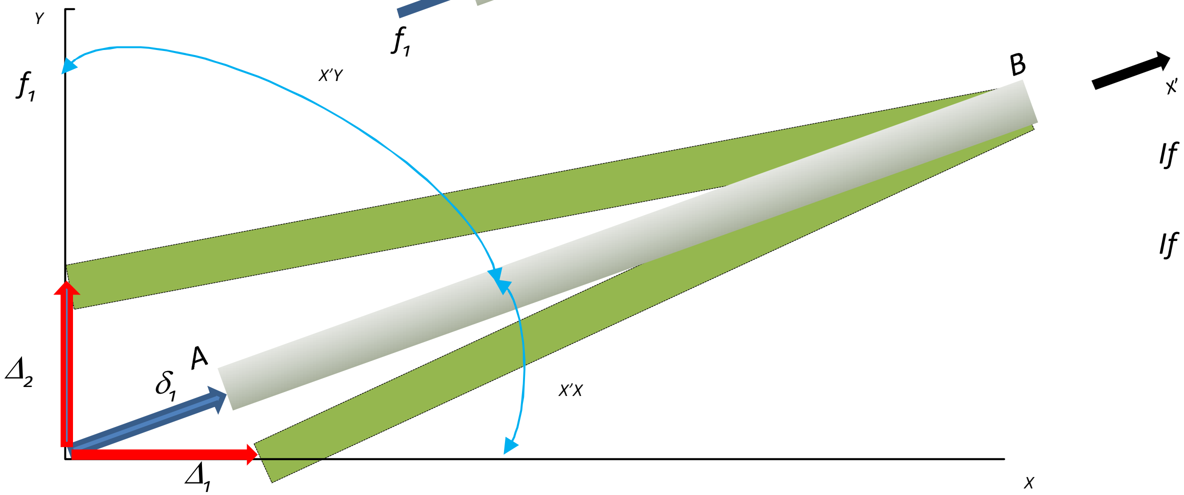
$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \boxed{\frac{AE}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \xrightarrow{\text{symbolically}} \{f_1\} = [k]\{\delta_1\}$$

Element displacement vector

Element force vector

Element stiffness matrix




*If only* $\Delta_1$ *occurred* $\qquad \delta_1 = \Delta_1 \cos(X'X)$

*If only* $\Delta_2$ *occurred* $\qquad \delta_1 = \Delta_2 \cos(X'Y)$

*If both* $\Delta_1$ & $\Delta_2$ *occurred*

$$\delta_1 = \Delta_1 \cos(X'X) + \Delta_2 \cos(X'Y)$$

*Similar at end B*

$$\delta_2 = \Delta_3 \cos(X'X) + \Delta_4 \cos(X'Y)$$

*Writing in Matrix form:*

$$\delta_1 = \Delta_1 \cos(X'X)$$

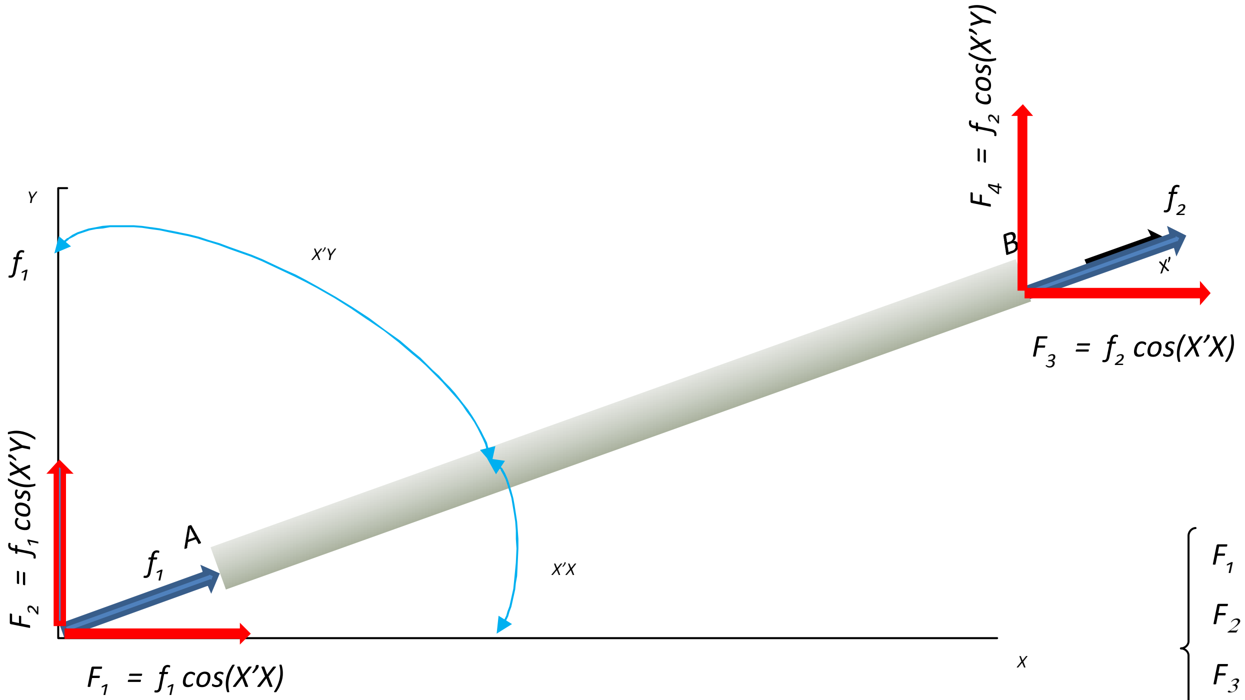
$$\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \begin{bmatrix} \cos(X'X) & \cos(X'Y) & 0 & 0 \\ 0 & 0 & \cos(X'X) & \cos(X'Y) \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{Bmatrix}$$

symbolically

$$\{\delta\} = [T]\{\Delta\}$$

Global to local transformation matrix

$$\{f_1\} = [k]\{\delta_1\} \qquad \{\delta\} = [T]\{\Delta\} \Rightarrow \boxed{\{f\} = [k][T]\{\Delta\}} \longrightarrow \{f\} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$




*Writing in Matrix form:*

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} \cos(X'X) & 0 \\ \cos(X'Y) & 0 \\ 0 & \cos(X'X) \\ 0 & \cos(X'Y) \end{bmatrix} \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$

$$\Downarrow$$

$$\{F\} = [T]^T \{f\}$$

*Finally, we write the global forces in terms of global displacement*

$$\{F\} = [T]^T [k] [T] \{\Delta\}$$

*Or*
$$\{F\} = [K] \{\Delta\}$$

Member global stiffness matrix

*Carry-out the matrix multiplications:*

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \frac{AE}{L} \begin{pmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ -\lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{pmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{Bmatrix} \qquad (14.16)$$

$$\lambda_x = cos(X'X)$$
$$\lambda_y = cos(X'Y)$$

EXAMPLE 14.2.1

Analyze truss $ABCD$ using the displacement method.

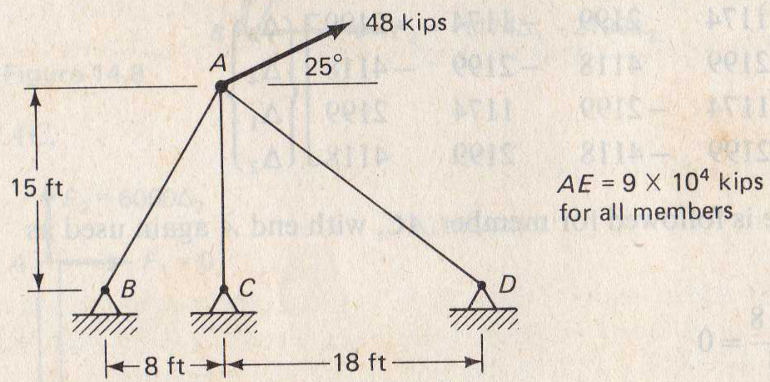



**Figure 14.6**

First we establish a global coordinate system. For this structure we will use traditional $x$, $y$ coordinates with origin at $B$. Then we assign subscripts serially from 1. The first ones are assigned to the *unknown* displacement components, after which subscripts are assigned to those $x$ and $y$ joint or support components that are constrained against motion. Thus the subscripting is as illustrated.

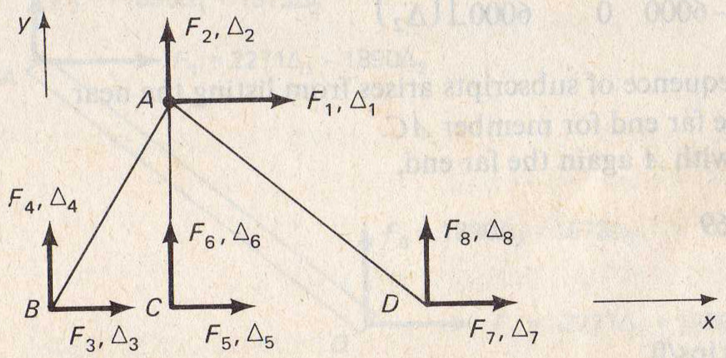



**Figure 14.7**

Since only joint $A$ can displace, only subscripts 1 and 2 pertain to displacements.

Now we develop the global force/displacement relations for each member, using Eq. (14.16). For member $AB$, letting $A$ be the far end, we use Eqs. (14.9) to get the direction cosines,

$$\cos(x'x) = \frac{8-0}{17} = 0.471$$

(X far – X near)/L

$$\cos(x'y) = \frac{15-0}{17} = 0.882$$

(Y far – Y near)/L

and combine them with

$$\frac{AE}{L} = \frac{9 \times 10^4}{17} = 5294 \text{ kips/ft}$$

in Eq. (14.16). The order of the subscripts that follow may seem peculiar. It is the result of our listing the near end displacements first. Thus

$$\begin{Bmatrix} F_3 \\ F_4 \\ F_1 \\ F_2 \end{Bmatrix}_{AB} = \begin{bmatrix} 1174 & 2199 & -1174 & -2199 \\ 2199 & 4118 & -2199 & -4118 \\ -1174 & -2199 & 1174 & 2199 \\ -2199 & -4118 & 2199 & 4118 \end{bmatrix} \begin{Bmatrix} \Delta_3 \\ \Delta_4 \\ \Delta_1 \\ \Delta_2 \end{Bmatrix}$$

A similar sequence is followed for member $AC$, with end $A$ again used as the far end,

$$\cos(x'x) = \frac{8-8}{15} = 0$$

$$\cos(x'y) = \frac{15-0}{15} = 1$$

$$\frac{AE}{L} = \frac{9 \times 10^4 \text{ kips}}{15 \text{ ft}} = 6000 \text{ kips/ft}$$

$$\begin{Bmatrix} F_5 \\ F_6 \\ F_1 \\ F_2 \end{Bmatrix}_{AC} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 6000 & 0 & -6000 \\ 0 & 0 & 0 & 0 \\ 0 & -6000 & 0 & 6000 \end{bmatrix} \begin{Bmatrix} \Delta_5 \\ \Delta_6 \\ \Delta_1 \\ \Delta_2 \end{Bmatrix}$$

Again the peculiar sequence of subscripts arises from listing the near end first and then the far end for member $AC$.

Finally for $AD$, with $A$ again the far end,

$$\cos(x'x) = -0.769$$

$$\cos(x'y) = 0.640$$

$$\frac{AE}{L} = 3841 \text{ kips/ft}$$

$$\begin{Bmatrix} F_7 \\ F_8 \\ F_1 \\ F_2 \end{Bmatrix}_{AD} = \begin{bmatrix} 2271 & -1890 & -2271 & 1890 \\ -1890 & 1573 & 1890 & -1573 \\ -2271 & 1890 & 2271 & -1890 \\ 1890 & -1573 & -1890 & 1573 \end{bmatrix} \begin{Bmatrix} \Delta_7 \\ \Delta_8 \\ \Delta_1 \\ \Delta_2 \end{Bmatrix}$$

You are urged to write the expressions for the direction cosines for $AD$ and satisfy yourself about the signs and values that evolved.

Although this is not usually a part of the matrix procedure, we will examine these member force/displacement relations pictorially. For $AB$,

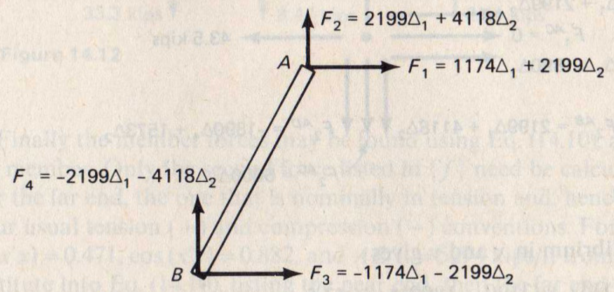



**Figure 14.8**

For $AC$,

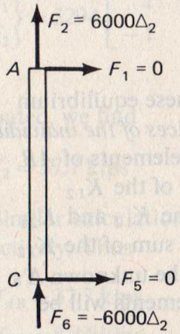



**Figure 14.9**

For $AD$,

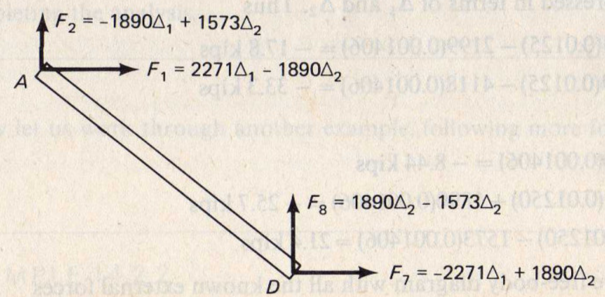



**Figure 14.10**

In all three, contributions to the forces from $\Delta_3$ through $\Delta_8$ were omitted since each of those displacements is zero.

Now we draw the free-body diagram of joint $A$, including the equal, opposite of each of the forces at end $A$ of each member, as well as the original externally applied 48-kip force that acts on joint $A$ but is shown in its $x$ and $y$ components of 43.5 kips and 20.3 kips, respectively.

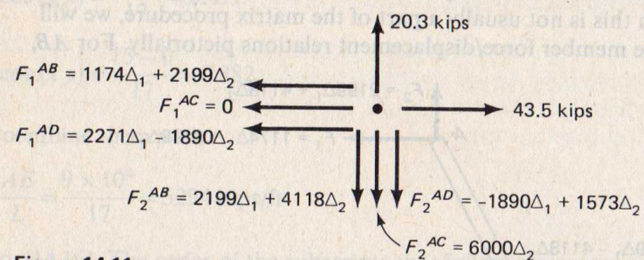



**Figure 14.11**

Force equilibrium in $x$ and $y$ gives

$$(1174 + 2271)\Delta_1 + (2199 - 1890)\Delta_2 = 43.5$$

and

$$(2199 - 1890)\Delta_1 + (4118 + 6000 + 1573)]\Delta_2 = 20.3$$

for which $\Delta_1 = 0.01250$ ft and $\Delta_2 = 0.001406$ ft.

Notice that *the coefficients* for $\Delta_1$ and $\Delta_2$ found in these equilibrium equations *are sums of elements in the global stiffness matrices of the individual members.* Specifically, $1174 + 2271$ is the sum of the $K_{11}$ elements of $AB$ and $AC$ (there is no $K_{11}$ for $AC$); $2199 - 1890$ is the sum of the $K_{12}$ elements and also of the $K_{21}$ elements for $AB$ and $AD$ (the $K_{12}$ and $K_{21}$ elements for $AC$ are zero); and $4118 + 6000 + 1573$ is the sum of the $K_{22}$ elements for all three members. That the coefficients for the unknown $\Delta$'s can be generated directly by summing stiffness matrix elements will be used to advantage in solutions henceforth.

Now that the unknown displacements have been determined, the support reactions can be calculated. They are forces $F_3$ through $F_8$, each having been expressed in terms of $\Delta_1$ and $\Delta_2$. Thus

$$F_3 = -1174(0.0125) - 2199(0.001406) = -17.8 \text{ kips}$$

$$F_4 = -2199(0.0125) - 4118(0.001406) = -33.3 \text{ kips}$$

$$F_5 = 0$$

$$F_6 = -6000(0.001406) = -8.44 \text{ kips}$$

$$F_7 = -2271(0.01250) + 1890(0.001406) = -25.7 \text{ kips}$$

$$F_8 = 1890(0.01250) - 1573(0.001406) = 21.4 \text{ kips}$$

The structure free-body diagram with all the known external forces and support reactions is shown.

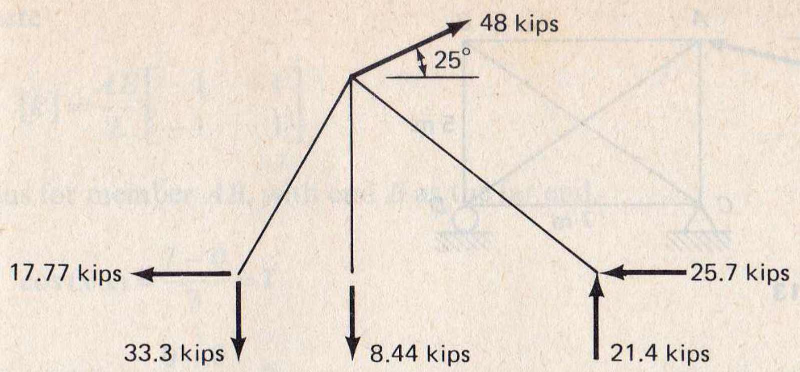



**Figure 14.12**

Finally the member forces may be found using Eq. (14.10), applied for each member. Only the second force listed in $\{f\}$ need be calculated. That is for the far end, the one that is nominally in tension and, hence, adheres to our usual tension $(+)$ and compression $(-)$ conventions. For $AB$, with $\cos(x'x) = 0.471$, $\cos(x'y) = 0.882$, and $AE/L = 5294$ kips/ft from before, we substitute into Eq. (14.10), listing the near end, then the far end displacements in $\{\Delta\}$.

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = 5294 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0.471 & 0.882 & 0 & 0 \\ 0 & 0 & 0.471 & 0.882 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.0125 \\ 0.00146 \end{Bmatrix}$$

Evaluated, we find

$$f_2 = 37.7 \text{ kips}$$

Similar calculations for $AC$ and $AD$ give $+8.44$ kips and $-33.5$ kips, respectively. Thus

$$f_{AB} = 37.7 \text{ kips}$$

$$f_{AC} = 8.44 \text{ kips}$$

$$f_{AD} = -33.5 \text{ kips}$$

completing the analysis.

ขอบคุณ และขอให้โชคดีทุกคน

ส.ค.ส. 2552

จากรศ. สิริวัฒน์ ไชยชนะ