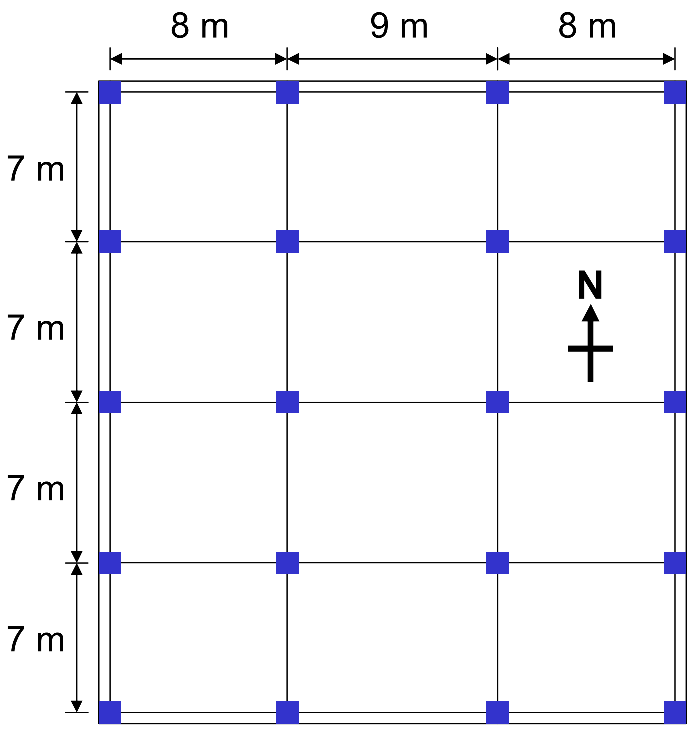
Design Example: A simple two-way post-tensioned plate for a residential high-rise building



Layout of a typical residential floor plate.

STEP 1 : Define Loads

D.L. = self-weight of the structure

Super imposed D.L.:

= 120 kg/m^2 for partitions

L.L. = 200 kg/m^2 (residential)

STEP 2 : Define Materials

Normal weight concrete = 2,400 kg/m^3

f'_c (28 days) = 350 ksc

Compressive strength when post-tensioned (typically after 24-hours)

f'_{ci} = 210 ksc

Mild reinforcing steel f_y = 4,000 ksc

Post-tensioning steel = unbonded tendons, ½"-diameter, 7-wire strands

A = 0.153 in^2 = 0.987 cm^2, f_{pu} = 270 ksi = 19,000 ksc

Estimated pre-stress losses = 15 ksi = 1,056 ksc (ACI318 Section 18.6)

f_{se} = 0.7 f_{pu} = 0.7(19,000) – 1,056 = 12,244 ksc (ACI318 Section 18.5.1)

$P_{eff} = A \times f_{se} = 0.987 \times 12,244 / 1,000$ = 12.085 ton/tendon

STEP 3 : Determine the preliminary slab thickness h

Start with L/h = 45 where the longest span = 9 m = 9 x 100 = 900 cm

h = 900 / 45 = 20 cm is the preliminary plate (slab) thickness.

D.L. = self weight = 0.2 x 2,400 = 480 kg/m^2

STEP 4 : Design the East-West interior frame

Use the Equivalent Frame Method of ACI 318 Section 13.7 (Exclude 13.7.7.4-5)

Total bay width between centerlines = 7 m

Ignore the column stiffness for simplicity of hand calculations.

No pattern loading required, since LL/DL = 200/480 < 3/4 (ACI318 Section 13.7.6)

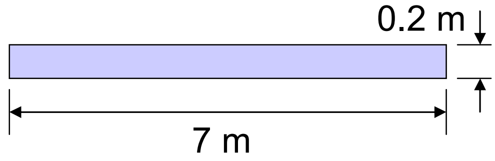
STEP 5 : Section Properties

Two-way plate must be designed as class U (ACI 318 Section 18.3.3).

The gross sectional properties are allowed (ACI 318 Section 18.3.4).

$A = bh = (700\ \text{cm})(20\ \text{cm}) = 14{,}000\ \text{cm}^2$

$S = bh^2/6 = (700\ \text{cm})(20\ \text{cm})^2 / 6 = 46{,}667\ \text{cm}^3$



STEP 6 : Design Parameters

Allowable stresses: Class U (ACI 318 Section 18.3.3)

Strength at time of jacking f'_{ci} = 210 ksc

Compression = $0.60 f'_{ci} = 0.6\,(210\ \text{ksc}) = 126$ ksc

Tension = $0.795\sqrt{f'_{ci}} = 0.795\,(210)^{0.5} = 11.5$ ksc

At service loads (ACI 318 Section 18.4.2(a) and 18.3.3)

Compressive strength f'_c = 350 ksc

Compression = $0.45 f'_c = 0.45\,(350\ \text{ksc}) = 158$ ksc

Tension = $1.59\sqrt{f'_{ci}} = 1.59\,(350)^{0.5} = 29.8$ ksc

Average pre-compression limits (ACI 318 Section 18.12.4)

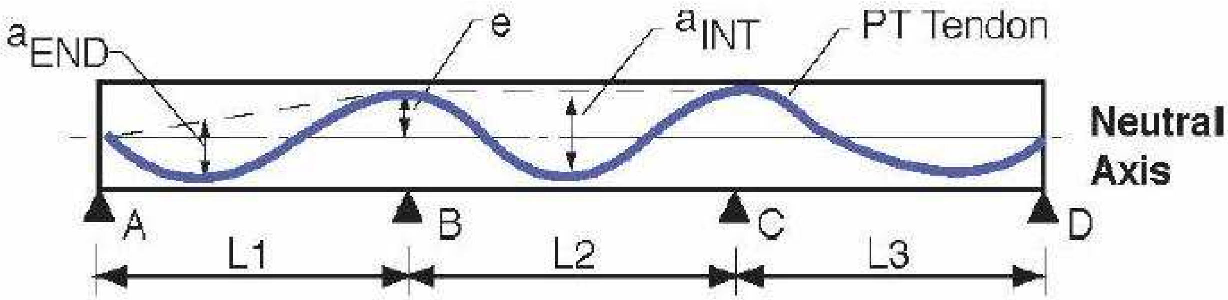
P / A = 8.8 ksc minimum to 21 ksc maximum

Target load balances, use 60% - 80% of DL (self-weight) for plate

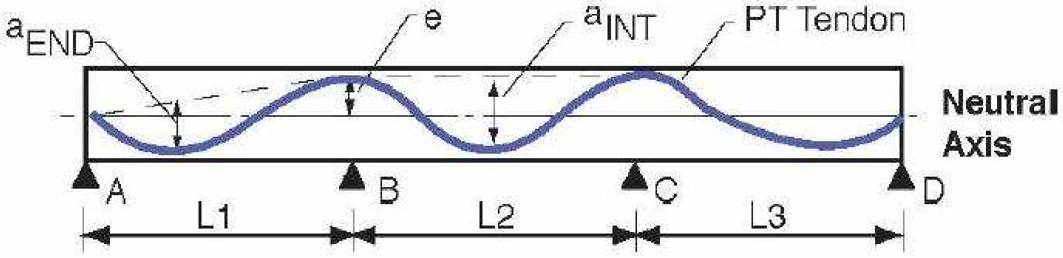
For this example: 0.75 DL = 0.75 (480) = 360 kg/m^2

STEP 7 : Tendon Profile

Use the parabolic shape. For a layout with spans of similar length, the tendons will be typically located at the highest allowable point at the interior columns, the lowest possible point at the mid-spans, and at the neutral axis at the anchor locations. This provides the maximum drape for load-balancing.



The continuous post-tensioned beam.



The continuous post-tensioned beam.

Tendon Ordinate	Tendon (CG) Location*
Exterior support - anchor	10 cm
Interior support - top	17.5 cm
Interior span - bottom	2.5 cm
End span - bottom	4.5 cm

(CG) means center of gravity, *Means measured from slab bottom

a_{INT} = 17.5 – 2.5 = 15 cm

a_{END} = (10 + 17.5)/2 – 4.5 = 9.25 cm

STEP 8 : Prestress force P required to balance 75% DL

$w_b = 0.75\, w_{DL} = 0.75\ (480\ kg/m^2)\ (7\ m)\ /\ 1{,}000 = 2.52\ t/m$

Since the spans are similar length, the end span will typically govern the maximum required post-tensioning force.

This is due to the significantly reduced tendon drape, **a_{END}**.

The force **P** needed in the tendons to counteract the load in the end bay is,

$P = (w_b L^2 / 8) / a_{END} = (2.52 \times 8^2 /8) / 0.0925 = 218$ tons

STEP 9 : Check pre-compression allowance

Determine the number of tendons required to attain the 218 tons,

Number of tendons = (218 tons) / (12.085 tons/tendon) = 18.04 **USE 18 tendons**

Actual force for banded tendons = (18 tendons) (12.085 tons) = 217.5 tons

The balanced load for the end span is slightly adjusted,

$w_b = (217.5 / 218)\ (2.52\ t/m) = 2.51\ t/m$

Determine the actual pre-compression stress in the slab,

$P_{actual} / A = (217.5 \times 1{,}000) / (14{,}000\ cm^2) = 15.5$ ksc (> 8.8 ksc minimum and < 21 ksc maximum) **OK**

STEP 10 : Check the interior span force P

$P = (w_b L^2 / 8) / a_{INT} = (2.52 \times 9^2 /8) / 0.15 = 170$ tons < 217.5 tons

Therefore, a smaller force is required in the center bay.

For this example, continue the force required for the end spans into the interior span and check the amount of load that will be balanced:

$w_b = 217.5 \times 8 \times 0.15 / 9^2 = 3.22$ t/m

Therefore, $w_b / w_{DL} = 3.22 / 3.36 = 96\% < 100\%$ **OK**

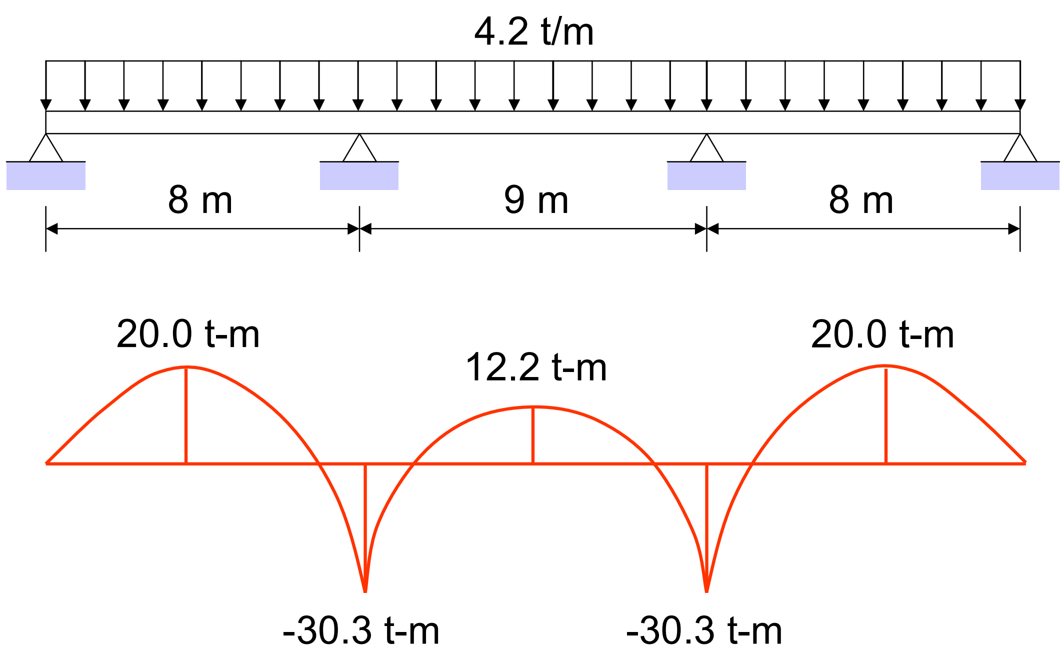
Therefore, for the East-West interior frame:

Effective prestress force, P_{eff} = 217.5 tons

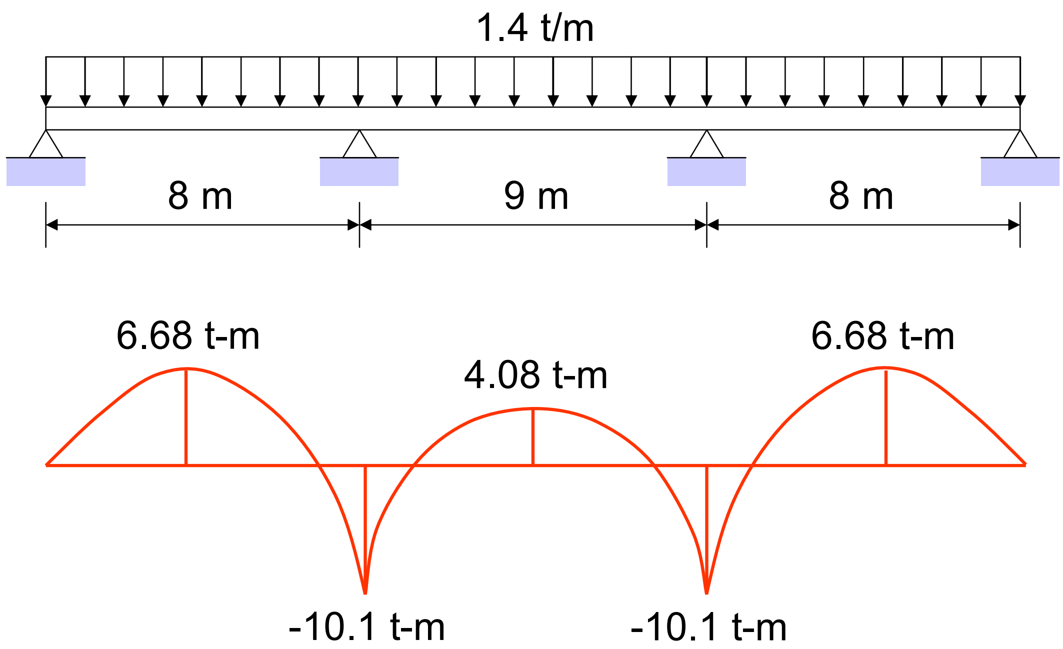
STEP 11 : ตรวจสอบหน่วยแรงในแผ่นพื้น

คำนวณโมเมนต์บวกและลบมากที่สุดจากน้ำหนักบรรทุกคงที่ น้ำหนักจร และน้ำหนักถ่วงสมดุล ผลร่วมกระทำของน้ำหนักเหล่านี้จะนำไปคำนวณหน่วยแรงในแผ่นพื้นขณะอัดแรงและรับน้ำหนักใช้งาน

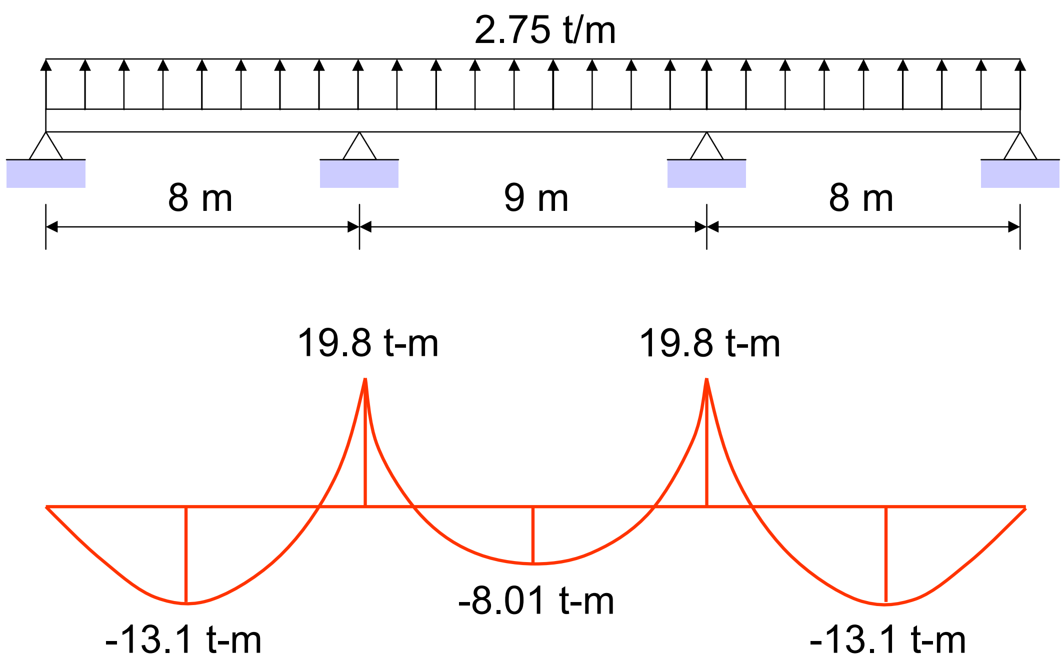
Dead Load Moments: $w_{DL} = (480+120)(7\ m) / 1{,}000 = 4.2$ t/m



Live Load Moments: w_{LL} = (200)(7 m) / 1,000 = 1.4 t/m



Total Balancing Moments, M_{bal} : w_b = 2.75 t/m ↑ (average of 3 bays)



STEP 12 Stage 1 : Stresses immediately after jacking (DL+PT)

ACI 318 18.4.1

Mid-span stresses, $f_{top} = (-M_{DL} + M_{bal})/S – P/A$

$f_{bot} = (+M_{DL} - M_{bal})/S – P/A$

Interior Span:

f_{top} = (-12.2 + 8.01)(100)(1,000)/46,667 – 15.5 ksc

= -9.0 – 15.5 = -24.5 ksc compression < 0.60f'_{ci} = 126 ksc **OK**

f_{bot} = (+12.2 - 8.01)(100)(1,000)/46,667 – 15.5 ksc

= 9.0 – 15.5 = -6.5 ksc compression < 0.60f'_{ci} = 126 ksc **OK**

End Span:

f_{top} = (-20.0 + 13.1)(100)(1,000)/46,667 – 15.5 ksc

= -14.8 – 15.5 = -30.3 ksc compression < 0.60f'_{ci} = 126 ksc **OK**

f_{bot} = (+20.0 – 13.1)(100)(1,000)/46,667 – 15.5 ksc

= 14.8 – 15.5 = -0.2 ksc compression < 0.60f'_{ci} = 126 ksc **OK**

Support stresses, $f_{top} = (+M_{DL} - M_{bal})/S – P/A$

$f_{bot} = (-M_{DL} + M_{bal})/S – P/A$

f_{top} = (30.3 – 19.8)(100)(1,000)/46,667 – 15.5 ksc

= 22.5 – 15.5 = 7.0 ksc tension < $0.8\sqrt{f'_{ci}}$ = 11.6 ksc **OK**

f_{bot} = (-30.3 + 19.8)(100)(1,000)/46,667 – 15.5 ksc

= -22.5 – 15.5 = -38 ksc compression < 0.60f'_{ci} = 126 ksc **OK**

Stage 2 : Stresses at service load (DL+LL+PT) ACI 318 18.3.3 & 18.4.2

Mid-span stresses, $f_{top} = (-M_{DL} - M_{LL} + M_{bal})/S – P/A$

$f_{bot} = (+M_{DL} + M_{LL} - M_{bal})/S – P/A$

Interior Span:

f_{top} = (-12.2 – 4.08 + 8.01)(100)(1,000)/46,667 – 15.5 ksc

= -17.7 – 15.5 = -33.2 ksc compression < 0.45f'$_c$ = 157.5 ksc **OK**

f_{bot} = (+12.2 + 4.08 - 8.01)(100)(1,000)/46,667 – 15.5 ksc

= 17.7 – 15.5 = 2.2 ksc tension < 1.6 √ f'$_c$ = 29.9 ksc **OK**

End Span:

f_{top} = (-20.0 – 6.68 + 13.1)(100)(1,000)/46,667 – 15.5 ksc

= -29.1 – 15.5 = -44.6 ksc compression < 0.45f'$_c$ = 157.5 ksc **OK**

f_{bot} = (+20.0 + 6.68 – 13.1)(100)(1,000)/46,667 – 15.5 ksc

= 29.1 – 15.5 = 13.6 ksc tension < 1.6 √ f'$_c$ = 29.9 ksc **OK**

Support stresses, $f_{top} = (M_{DL} + M_{LL} - M_{bal})/S – P/A$

$f_{bot} = (-M_{DL} - M_{LL} + M_{bal})/S – P/A$

f_{top} = (30.3 + 10.1 – 19.8)(100)(1,000)/46,667 – 15.5 ksc

= 44.1 – 15.5 = 28.6 ksc tension < 1.6 √ f'$_c$ = 29.9 ksc **OK**

f_{bot} = (-30.3 – 10.1 + 19.8)(100)(1,000)/46,667 – 15.5 ksc

= -44.1 – 15.5 = -59.6 ksc compression < 0.45f'$_c$ = 157.5 ksc **OK**

All stresses are within the permissible code limits.

STEP 13 : Determine the factored moments

Primary post-tensioning moments M_{prim} vary along the length of span

$$M_{prim} = P \times e$$

where e = 0 cm at exterior support and
e = 7.5 cm at interior support

$M_{prim} = 217.5 \times 7.5 / 100 = 16.3$ t-m

Secondary post-tensioning moments M_{sec} vary linearly between supports

$M_{sec} = M_{bal} - M_{prim} = 19.8 - 16.3 = 3.5$ t-m at interior supports

Load combination : $M_u = 1.4\ M_{DL} + 1.7\ M_{LL} + 1.0\ M_{sec}$

At midspan, $M_u = 1.4(20.0) + 1.7(6.68) + 1.0(1.75) = 41.1$ t-m

At support, $M_u = 1.4(-30.3) + 1.7(-10.1) + 1.0(3.5) = -56.1$ t-m

STEP 14 : Determine minimum bonded reinforcement

Positive moment region:

Interior span: $f_t = 2.2$ ksc $< 0.53 \sqrt{f'_c} = 0.53 \sqrt{350} = 9.92$ ksc

No positive reinforcement is required (ACI 318 18.9.3.1)

Exterior span: $f_t = 13.6$ ksc $> 0.53 \sqrt{f'_c} = 0.53 \sqrt{350} = 9.92$ ksc

Minimum positive reinforcement is required (ACI 318 18.9.3.2)

$y = f_t / (f_t + f_c)\ h = [13.6 / (13.6 + 44.6)](20\text{ cm}) = 4.67$ cm

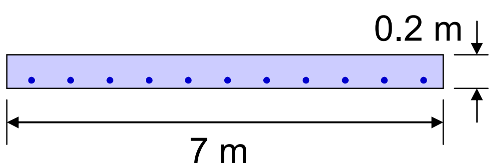
$N_c = M_{DL+LL} / S \times 0.5\ y\ L_2$

$= [(20.0+6.68)(100)/46,667](0.5)(4.67)(700) = 93.5$ tons

$A_{s,min} = N_c / 0.5\ f_y = (93.5\times1,000) / [0.5(4,000\text{ ksc})] = 46.75\text{ cm}^2$

ใส่เหล็กกระจายสม่ำเสมอตลอดความกว้างพื้น 7 เมตร ใกล้ผิวรับแรงดึง

$A_{s,min} = (46.75\text{ cm}^2) / (7\text{ m}) = 6.68\text{ cm}^2 / \text{m}$



USE DB16 @ 0.30 m at bottom = 6.70 cm²/m

Negative moment region:

$A_{s,min}$ = 0.00075 A_{cf} (as per ACI 318 18.9.3.3)

At interior support:

A_{cf} = max of (20 cm)[(9 m+8 m)/2 or 7 m](100) = 14,000 cm^2

$A_{s,min}$ = 0.00075 (14,000 cm^2) = 10.5 cm^2

USE 10 DB12 at the top (11.31 cm^2)

At exterior support:

A_{cf} = max of (20 cm)[(8 m)/2 or 7 m](100) = 8,000 cm^2

$A_{s,min}$ = 0.00075 (8,000 cm^2) = 6.0 cm^2

USE 6 DB12 at the top (6.79 cm^2)

STEP 15 : Check minimum reinforcement to see if it is sufficient for ultimate strength

$M_n = (A_s f_y + A_{ps} f_{ps}) (d - a/2)$

A_{ps} = 0.987 $cm^2 \times$ (18 tendons) = 17.77 cm^2

$f_{ps} = f_{se} + 704 + f'_c bd / (300A_{ps})$ for slabs with L/h > 35 (Sec.18.7.2)

= 12,244 + 704 + [(350 ksc)(700 cm) d] / (300 $\times$ 17.77)

= 12,948 + 46 d

$a = (A_s f_y + A_{ps} f_{ps}) / (0.85 f'_c b)$

At supports: d = 17.5 cm

f_{ps} = 12,948 + 46 (17.5) = 13,753 ksc

a = [(11.31 cm^2)(4,000 ksc) + (17.77 cm^2)(13,753 ksc)] / (0.85$\times$350$\times$700)

= 1.39 cm

$$M_n = (A_s f_y + A_{ps} f_{ps})(d - a/2)$$

$$= [(11.31\ cm^2)(4{,}000\ ksc) + (17.77\ cm^2)(13{,}753\ ksc)]\ (17.5 - 1.39/2)$$

$$= 4{,}867{,}246\ kg\text{-}cm = 48.67\ t\text{-}m$$

$\phi M_n = 0.9\ (48.67) = 43.8\ t\text{-}m < [\ M_u = 56.1\ t\text{-}m\ ]$ **NG**

Therefore, reinforcement for ultimate strength requirements governs.

Try $A_{s,reqd} = (56.1 / 43.8)(11.31) = 14.47\ cm^2$ **USE 30 DB12 (33.9 cm²)**

$$a = [(\mathbf{33.9}\ cm^2)(4{,}000\ ksc) + (17.77\ cm^2)(13{,}753\ ksc)] / (0.85 \times 350 \times 700)$$

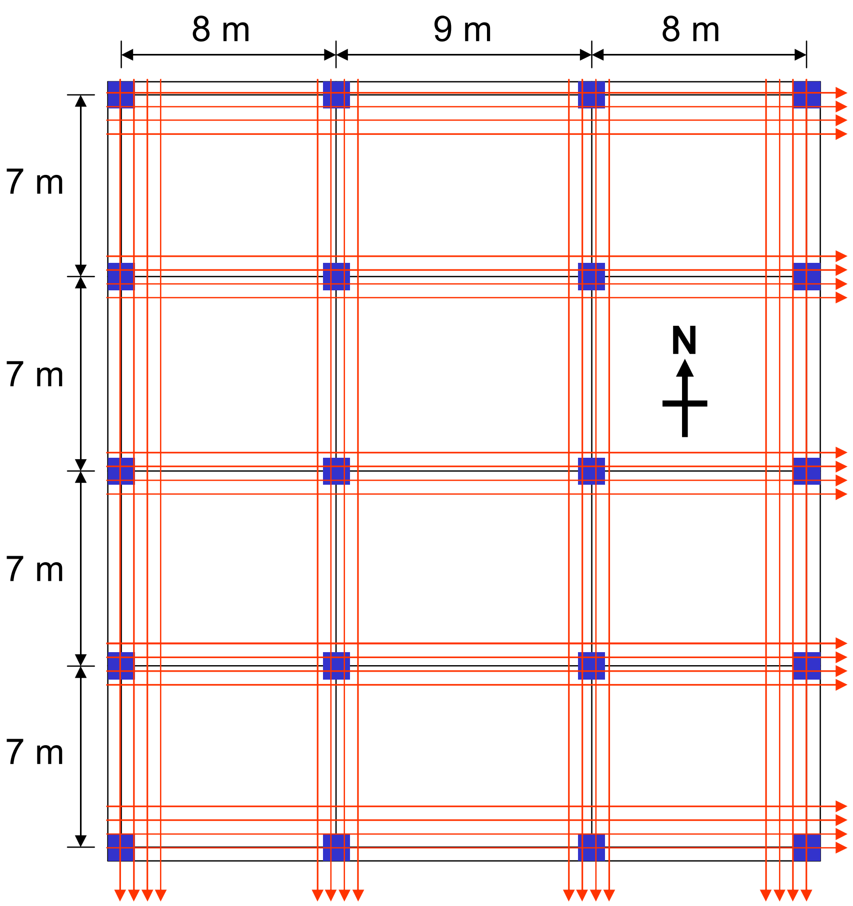
$$= \mathbf{1.83\ cm}$$

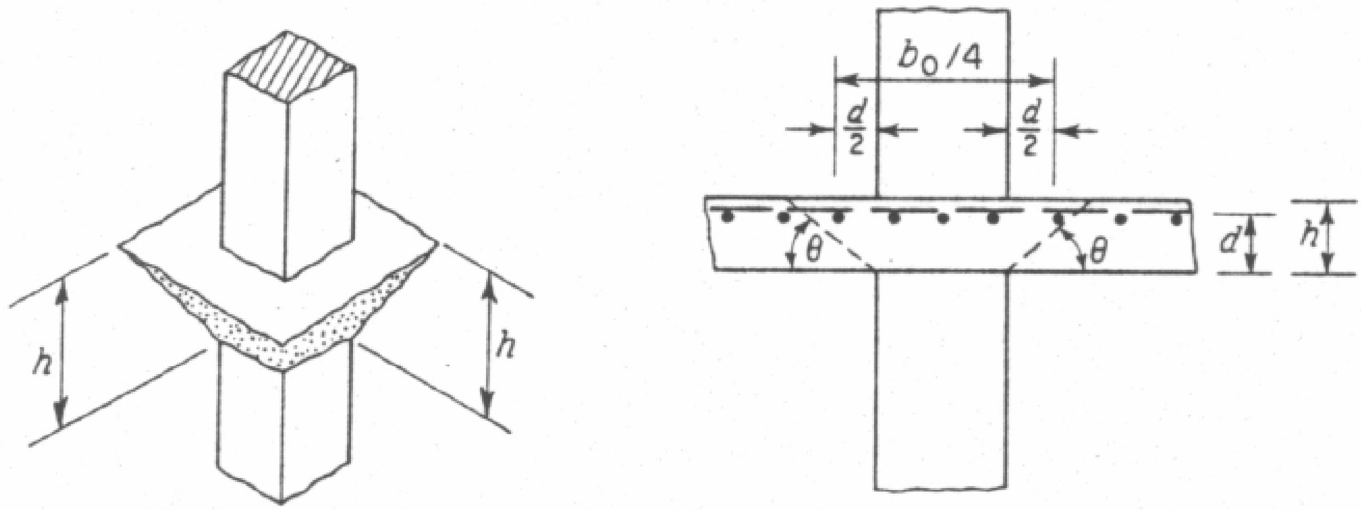
$$M_n = [(\mathbf{33.9}\ cm^2)(4{,}000\ ksc) + (17.77\ cm^2)(13{,}753\ ksc)]\ (17.5 - \mathbf{1.83}/2)$$

$$= 6{,}302{,}148\ kg\text{-}cm = 63.02\ t\text{-}m$$

$\phi M_n = 0.9\ (63.02) = 56.7\ t\text{-}m > [\ M_u = 56.1\ t\text{-}m\ ]$ **OK**

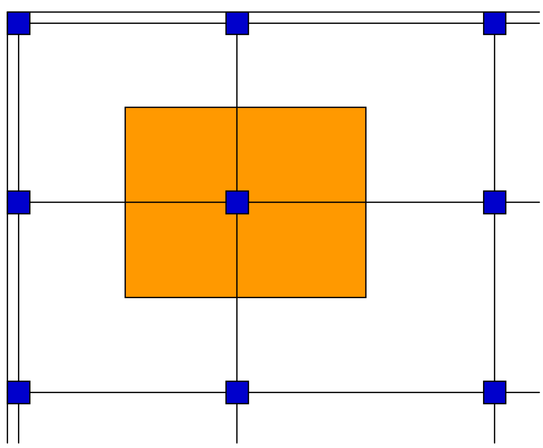
Final PT Layout





PUNCHING SHEAR FAILURE SURFACE

STEP 15 : Check Punching Shear



Interior column : 40 cm x 40 cm

Tributary area = (7 m) (8 m + 9 m) / 2

= 59.5 m²

At supports: $d = 17.5$ cm

Shear area = (0.4+0.175) x (0.4+0.175)

= 0.33 m²

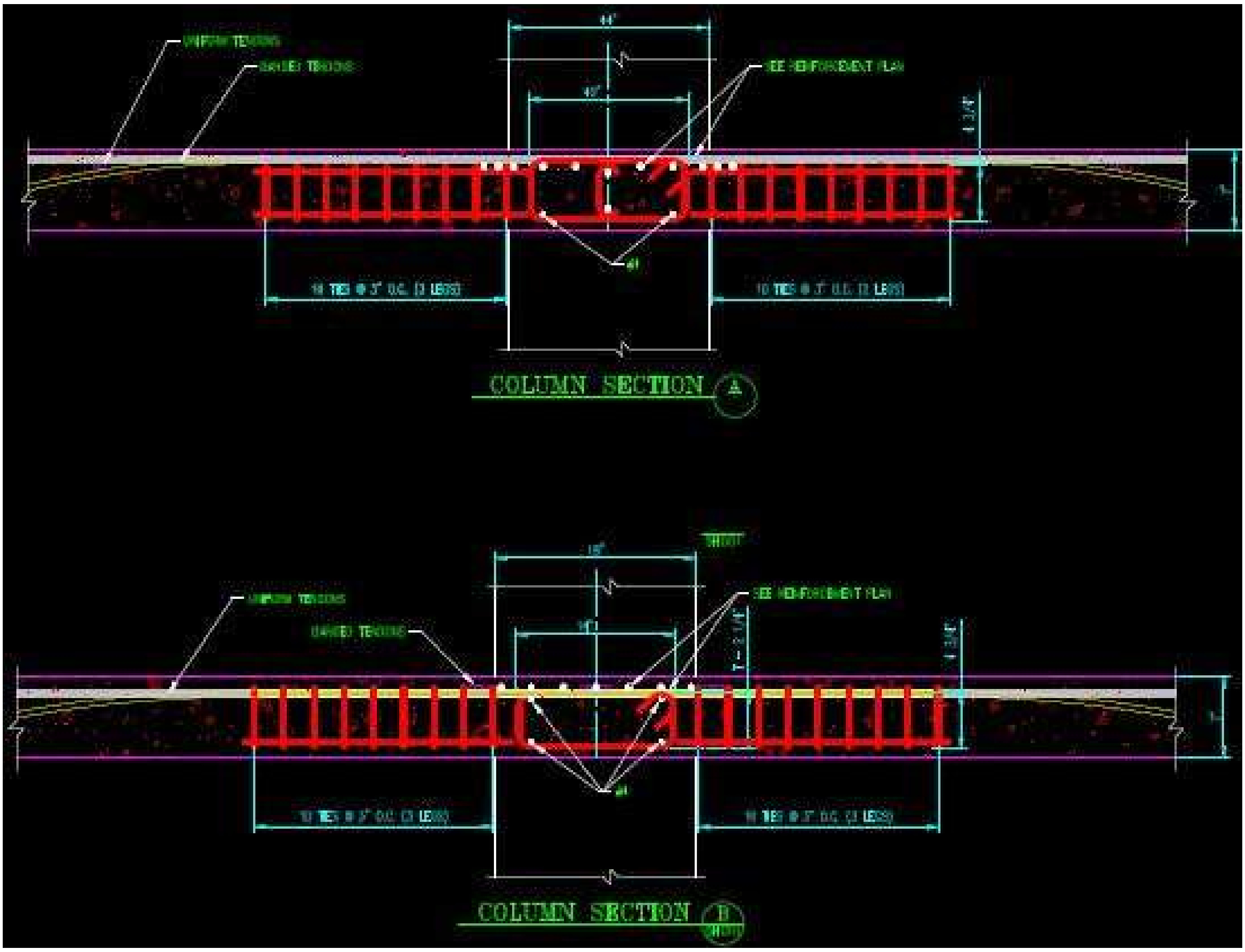
w_u = 1.4 (480+120) + 1.7 (200) = 1,180 kg/m²

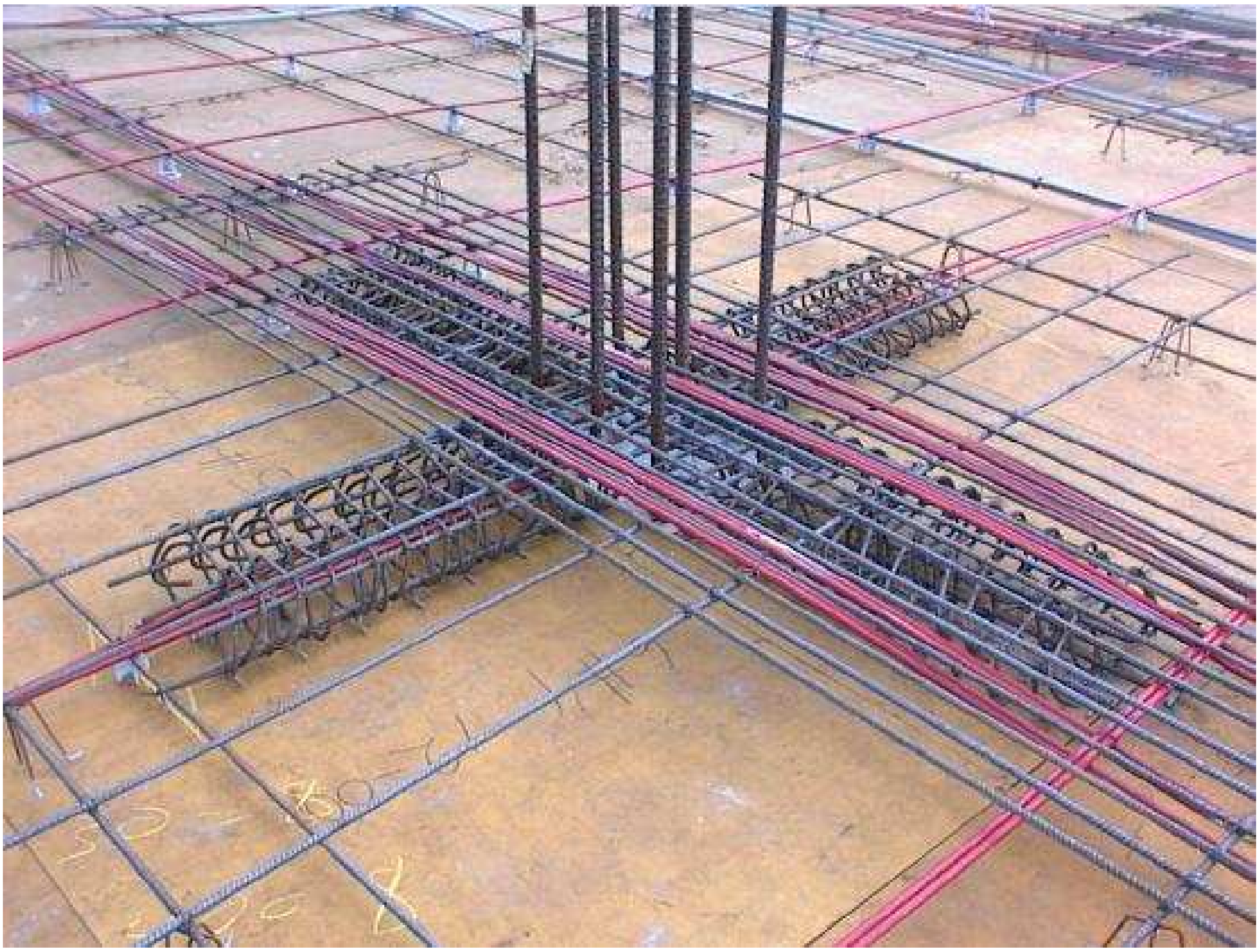
V_u = (1,180 kg/m²) (59.5 m² – 0.33 m²) / 1,000 = 69.8 tons

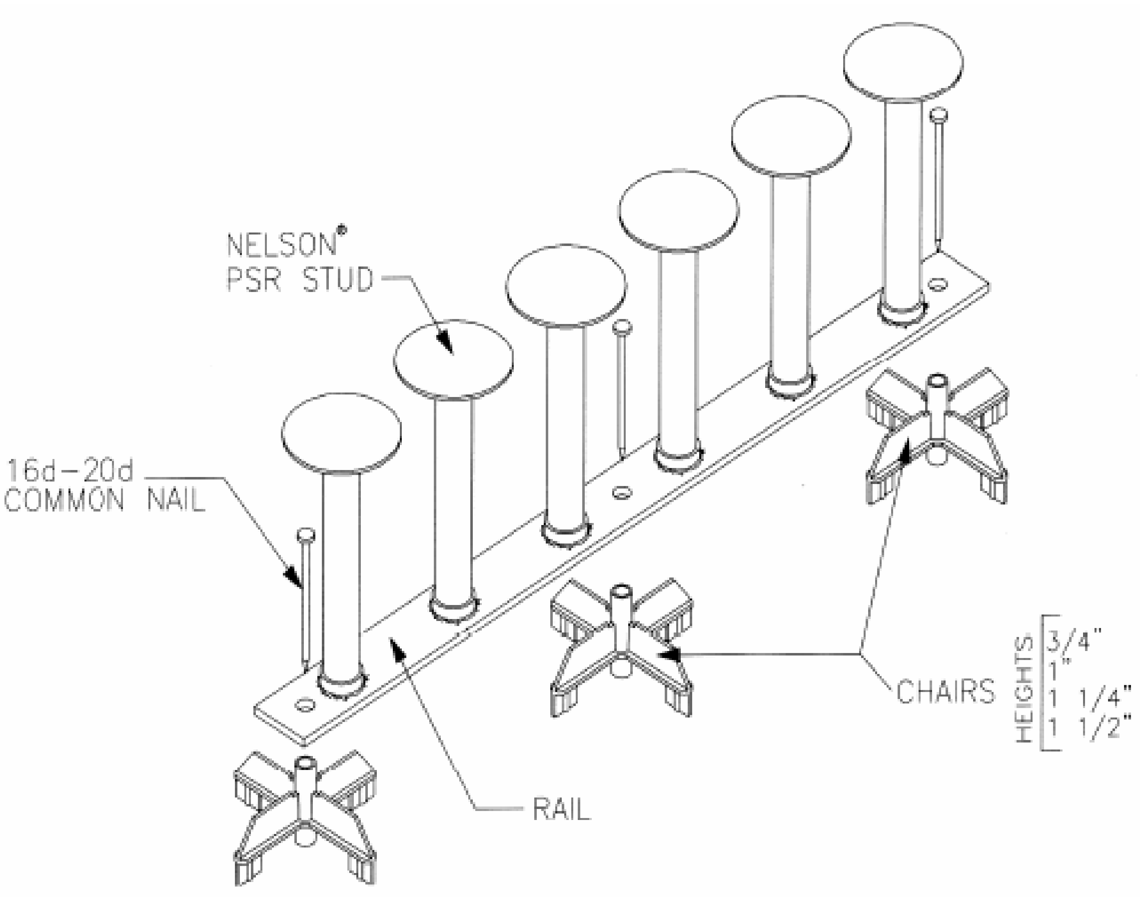
$$V_c = 1.06\sqrt{f_c'}\, b_0\, d = 1.06\sqrt{350}\,(4)(40+17.5)(17.5)/1{,}000$$

= 79.8 tons < [V_u/ϕ = 69.8/0.85 = 82.1 tons] **NG**

Need Punching Shear Reinforcement


UNIFORM TENDONS
BANDED TENDONS
SEE REINFORCEMENT PLAN
COLUMN SECTION
A
UNIFORM TENDONS
BANDED TENDONS
SEE REINFORCEMENT PLAN
COLUMN SECTION
B




NELSON®
PSR STUD
16d-20d
COMMON NAIL
CHAIRS
HEIGHTS
3/4"
1"
1 1/4"
1 1/2"
RAIL