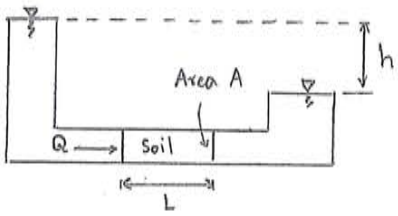
One dimensional flow (Ch. 14, L&W)

1.) Darcy's law



$$Q = k \cdot \frac{h}{L} A = kiA$$

Q =

k = coef. of permeability

i = hydraulic gradient = $\frac{h}{L}$

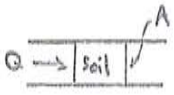
▷ valid for larminar flow

▷ not valid for turbulent flow

2.) Flow velocities

2.1) Approach velocity, v

$$v = \frac{Q}{A} = \frac{k \cdot i \cdot A}{A} = k \cdot i$$



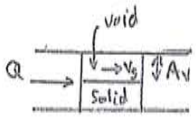
▷ neglect effect of void

2.2) Seepage velocity, v

$$Q = vA = v_s A_v$$

$$vA = v_s nA$$

$$v_s = \frac{v}{n}, \quad n = \text{porosity}$$



$$n = \frac{V_v}{V} = \frac{A_v}{A}$$

▷ consider effect of void & assumed straight void

Note: Both approach & seepage velocities is still fiziitious, why?

≠ actual velocities

3.) Heads

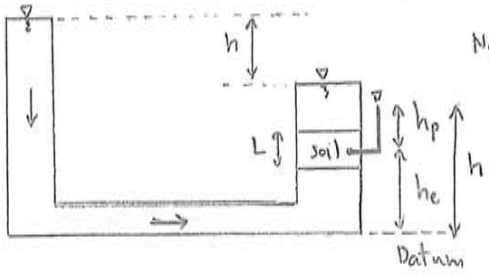
$$h = h_e + h_p$$

h = total head

h_e = elevation head

h_p = pore pressure head = $\frac{u}{\gamma_w}$

(the height water can rise in piezometric tube)



Notes: 1. velocity head in soil is negligible

2. All total head loss occurs in soil

3. Negative pore pressure can exist - above water table

4. Water flow is controlled only by total head

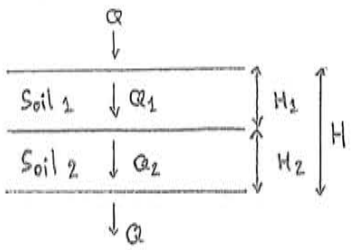
5. direction of flow - from higher total head to lower total head

6. Head calculations, $h = h_p + h_e$

hydraulic gradient, $i = \frac{\Delta h}{L} = \frac{(h_1 - h_2)}{L}$; Δh = difference of total head

4. Flow through layer soil

4.1 Vertical flow



Flow is the same

$$Q = Q_1 = Q_2$$

$$k_{ave} i A = k_1 i_1 A = k_2 i_2 A$$

$$k_{ave} i = k_1 i_1 = k_2 i_2 \qquad (1)$$

$$i_1 = \frac{k_{ave} i}{k_1} \quad ; \quad i_2 = \frac{k_{ave} \cdot i}{k_2}$$

Total head loss is the sum of each head loss

$$\Delta h = \Delta h_1 + \Delta h_2 \qquad i = \frac{\Delta h}{L} \qquad (2)$$

$$iH = i_1 H_1 + i_2 H_2$$

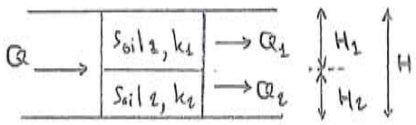
Substitute (1) in (2)

$$i \cdot H = \frac{k_{ave} i H_1}{k_1} + \frac{k_{ave} i H_2}{k_2}$$

$$H = \frac{k_{ave} H_1}{k_1} + \frac{k_{ave} H_2}{k_2} = k_{ave}\left(\frac{H_1}{k_1} + \frac{H_2}{k_2}\right)$$

Thus, $$k_{ave} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2}} = \frac{H}{\sum_{i=1}^{j}\left(\frac{H_i}{k_i}\right)}$$

4.2 Horizontal flow



same total head loss gradient

$$i = i_1 = i_2 \qquad (1)$$

total flow = sum of flow in each layer

$$Q = Q_1 + Q_2$$

$$k_{ave} i H = k_1 i_1 H_1 + k_2 i_2 H_2 \qquad (2)$$

Substitute (1) in (2)

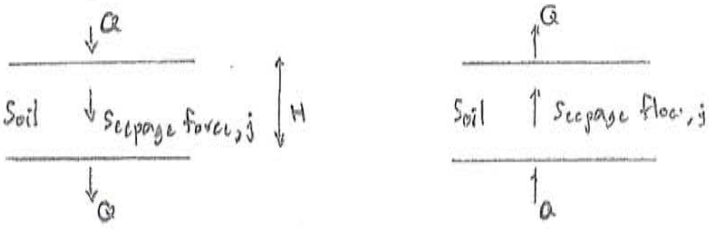
$$k_{ave} i H = k_1 i H_1 + k_2 i H_2$$

Thus $$k_{ave} = \frac{k_1 H_1 + k_2 H_2}{H} = \frac{\sum_{i=1}^{n}(k_i H_i)}{H}$$

5. Seepage force "water boundary force"

Seepage force / unit volume, $j = i \cdot \gamma_w$

Direction of j = direction of water flow, Q



6. Calculation of effective stress with water flow

$$\sigma' = \sigma - u \qquad \sigma' = \gamma_b \cdot z \pm j \begin{cases} \text{downward flow } + \\ \text{upward flow } - \end{cases}$$

σ = overbatic stress

$u = h_p \cdot \gamma_w$

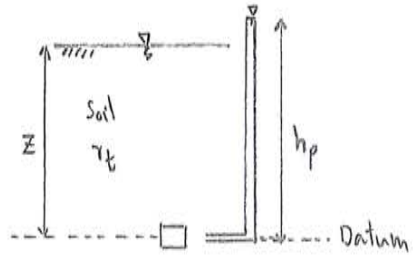
$h_p = h - h_e$

$h_p \neq h_{ps}$

7. Vertical effective stress by seepage force

$h_t = h_e + h_p$
$= Z + 0$
$= Z$

$h_t = h_e + h_p$
$= 0 + h_p$
$= h_p$



$\sigma_v' = \sigma_v - u$ at point 2

$= \gamma_t \cdot Z - h_p \cdot \gamma_w$

- No water flow ; $h_p = Z$

$$\sigma_v' = \gamma_t Z - Z\gamma_w = (\gamma_t - \gamma_w) Z = \gamma_b' Z$$

- Upward water flow : $h_p > Z$

$$i = \frac{\Delta h}{Z} = \frac{(h_p - Z)}{Z} = \frac{h_p}{Z} - 1 \dashrightarrow h_p = Z(1+i)$$

$$\sigma_v' = \gamma_t \cdot Z - h_p \gamma_w = \gamma_t Z - Z(i+1)\gamma_w = (\gamma_t - \gamma_w - i\gamma_w) Z = (\gamma_b - j) Z$$

- Downward water flow : $h_p < Z$

$$i = \frac{\Delta h}{Z} = \frac{(Z - h_p)}{Z} = 1 - \frac{h_p}{Z} \dashrightarrow h_p = Z(1-i)$$

$$\sigma_v' = \gamma_t Z - h_p \gamma_w = \gamma_t Z - Z(1-i)\gamma_w = (\gamma_t - \gamma_w + i\gamma_w) Z = (\gamma_b + j) Z$$

summary : $\sigma_v' = (\gamma_b \pm j) Z$

upward flow = $-j$; downward flow = $+j$

6. Quick condition

- only for cohesionless soil
- occur when $\sigma' = \sigma - u = 0$
- no strength

Situations :

1.) Earthquake
- loose sand

2.) Upward water flow

$$\sigma_v' = (\gamma_b - j)z = 0$$

$$j = \gamma_b$$

$$i_{cr} \cdot \gamma_w = \gamma_b$$

$$i_{cr} = \frac{\gamma_b}{\gamma_w}$$

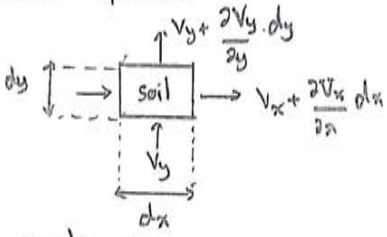
i_{cr} = critical hydraulic gradient causing quick condition

Usually, for sands, $\gamma_t \approx 2\gamma_w$

$$i_{cr} = \frac{\gamma_b}{\gamma_w} \approx \frac{(2\gamma_w - \gamma_w)}{\gamma_w} \approx 1 \quad ; \quad \gamma_b = \gamma_t - \gamma_w$$

<u>Two dimensional flow</u> (Ch. 18 L&W)

1. Partial differential equation



Continuity equation :

$$q_{in} = q_{out}$$

$$V_x \cdot dy(1) + V_y dx(1) = \left(V_x + \frac{\partial V_x}{\partial x} dx\right) dy \cdot (1) + \left(V_y + \frac{\partial V_y}{\partial y} dy\right) dx(1)$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \qquad (1)$$

Darcy's law

x-dir : $V_x = k_x i_x = k_x \cdot \frac{\partial h}{\partial x}$ (2)

y-dir : $V_y = k_y \cdot i_y = k_y \cdot \frac{\partial h}{\partial y}$ (3)

where i_x, i_y = hydraulic gradient in x-dir and y-dir, respectively

h = total head

k_x, k_y = permeability for flow in x-dir and y-dir, respectively

Substituting eqns. (2) & (3) in (1)

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right) = 0$$

for homogeneous property, k_x = constant, k_y = constant ($k_x \neq k_y$)

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \qquad (4)$$

∇h^2

for isotropic & homogeneous property, $k_x = k_y$ = constant

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad \text{laplace eqn.} \qquad (5)$$

Notes for eqn. (5)

1.) Meaning

change in i_x gradient per unit distance in x-dir

+ change " i_y " ———————— " y-dir

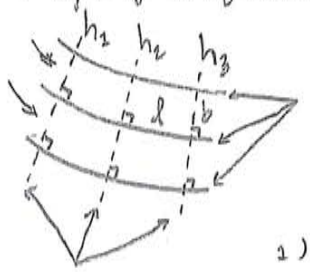
2.) Eqn (5) = Laplace eqn.

Solution is two series of line intersection at 90°

▷ called as "flow net"

- equipotential line
- flow line

2. Flow net for isotropic & homogeneous k



Equipotential line ≡ line with constant total head

flow line ≡ path of water flowing in the soil

Geometry of flow net

1.) Equipotential line ⊥ flow line ---> orthogonal net

2.) Net geometry, $r = \frac{b}{d}$ = const. ---> rectangular net

3.) For convenience, select $\frac{b}{d} = 1$ ---> square net

4.) No. of equipotential drop, n_d

5.) No. of flow path, n_f

6.) Equal head loss between adjacent equipotential lines

e.g. $h_1 - h_2 = h_2 - h_3$ = const. $= \frac{H}{n_d} = \Delta h$

<u>Calculation rate of water flow, q</u>

Each flow path ; $q = k\left(\frac{H}{n_d \cdot d}\right)b$; $q = kiA$ สำหรับ flow channal ที่ไม่เท่ากัน

All flow paths ; $q = k\left(\frac{H}{n_d \cdot d}\right)b n_f = k \cdot H\left(\frac{n_f}{n_d}\right)\left(\frac{b}{d}\right) = kHs$ สำหรับ flow channal ที่เท่ากัน

Note: shape factor, $s = \frac{n_f}{n_d}$ = independent of H & k

$Q_1 = Q_2$

$v \cdot A = v_s A_v$

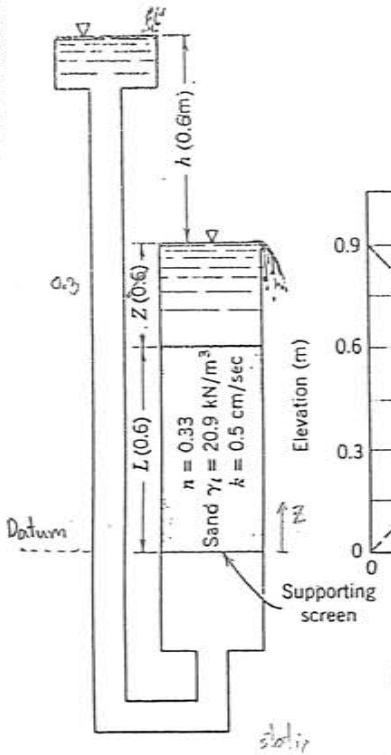
$vA = v_s (nA)$

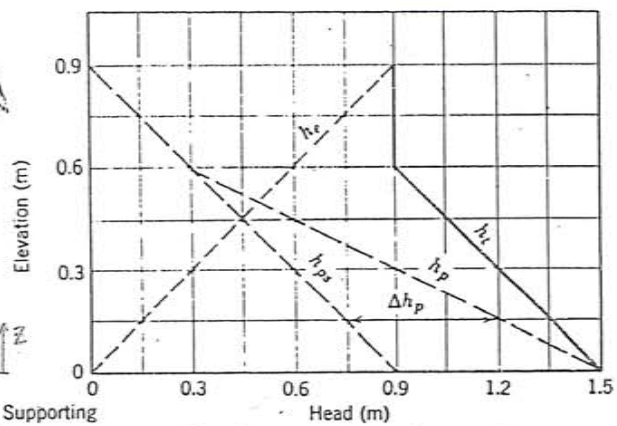
A minimum

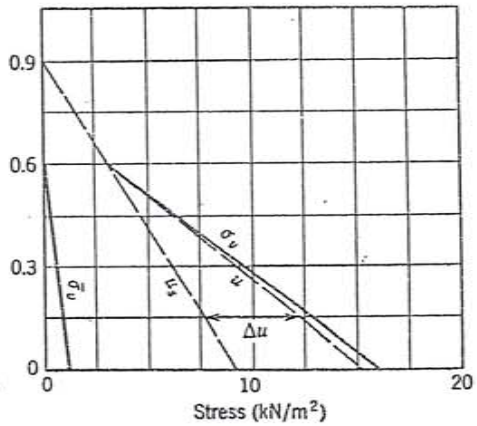
$v = v_s \cdot n$

$= \frac{k \cdot i}{n} \times n$

$= k \cdot i$







$i z = \frac{h}{L} \cdot z$

h_{ps} = static pore pressure head

$h_t' = h_t - i z$

Elev. (m.)	h_t (m)	h_e (m)	$h_p = h_t - h_e$ (m)	static h_{ps} (m)	$\Delta h_p = h_p - h_{ps}$ (m)
0.9	0.9	0.9	0	0	0
0.6	$1.5 - \frac{0.6}{0.6} \times 0.6 = 0.9$	0.6	0.3	0.3	0
0.3	$1.5 - \frac{0.6}{0.6} \times 0.3 = 1.2$	0.3	0.9	0.6	0.3
0	$h_e + h_p = 0 + (0.6 + 0.3 + 0.6) = 1.5$	0	1.5	0.9	0.6

Elevation (m)	$\Delta\sigma_v$ (kN/m²)	σ_v (kN/m²)	u (kN/m²)	$\bar{\sigma}_v$ (kN/m²)
0.9		0	0	0
	$0.3 \times \gamma_w = 2.94$			
0.6		2.94	$0.3 \times \gamma_w = 2.94$	0
	$0.6 \times \gamma_t = 12.54$			
0		15.48	$1.5\gamma_w = 14.72$	0.76

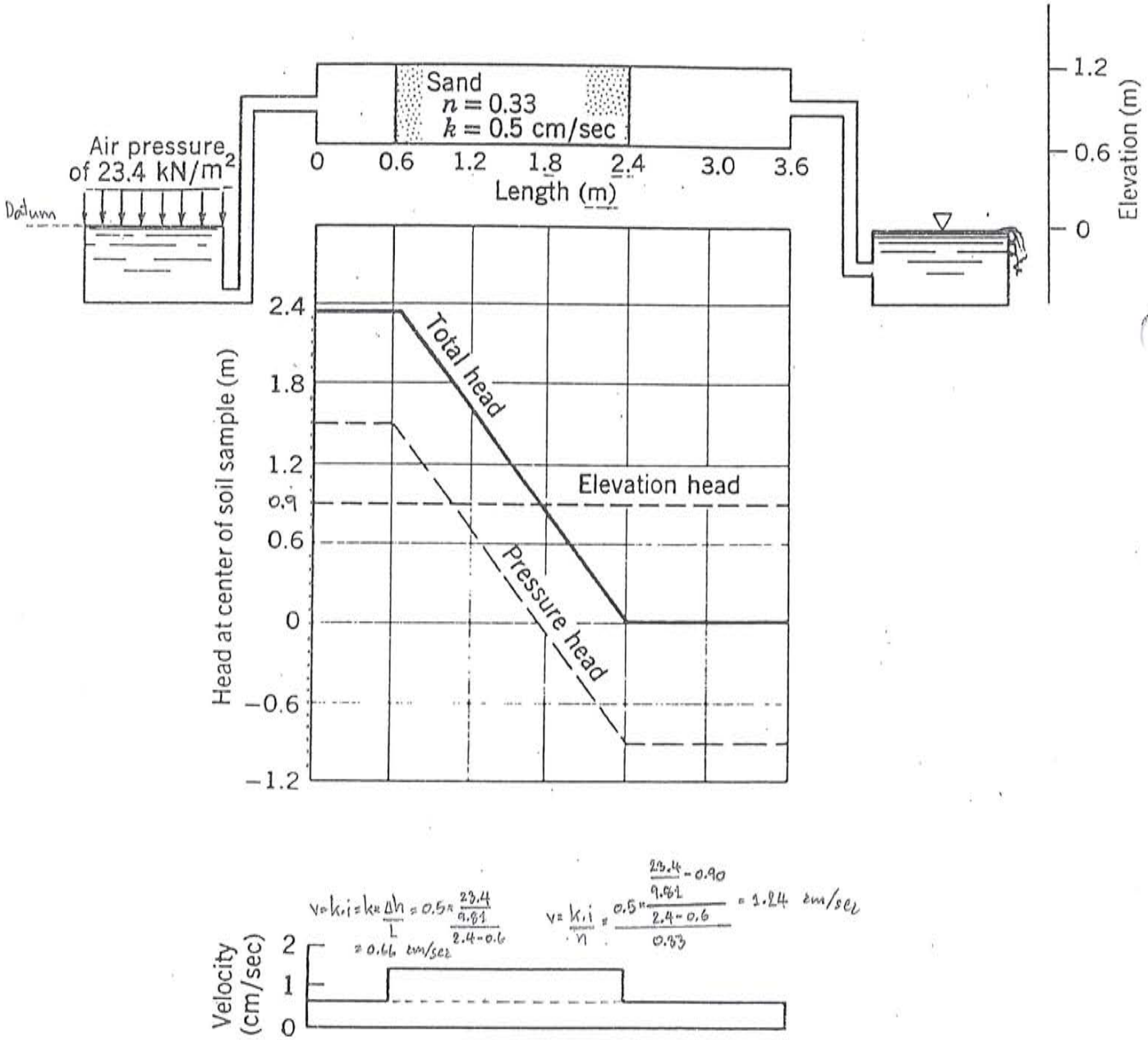
Fig. 17.12 Heads and stresses for fluid flow.

boundary water pressure

buoyancy water pressure (static)

pressure seepage

Elev. (m.)	σ_v (kN/m²)	$u = \gamma_w \cdot h_p$ (kN/m²)	$u_s = \gamma_w \cdot h_{ps}$ (kN/m²)	$\Delta u = u - u_s$ (kN/m²)	$\Delta\sigma_v$ (kN/m²)	$\sigma_v' = \sigma_v - u$ (kN/m²)
0.9	0	0	0	0	$\gamma_w(0.9-0.6) = 9.81 \times 0.3 = 2.94$	0
0.6	$\gamma_w \cdot (0.9-0.6) = 9.81 \times 0.3 = 2.94$	$9.81 \times 0.3 = 2.94$	$9.81 \times 0.3 = 2.94$	0	$\gamma_t \cdot (0.6-0.3) = 20.9 \times 0.3 = 6.27$	0
0.3	$\gamma_w(0.9-0.6) + \gamma_t(0.6-0.3) = 9.81 \times 0.3 + 20.9 \times 0.3 = 9.21$	$9.81 \times 0.9 = 8.83$	$9.81 \times 0.6 = 5.89$	2.94	$\gamma_t \cdot (0.3-0) = 20.9 \times 0.3 = 6.27$	0.38
0	$\gamma_w \cdot (0.9-0.6) + \gamma_t(0.6-0) = 9.81 \times 0.3 + 20.9 \times 0.6 = 15.48$	$9.81 \times 1.5 = 14.72$	$9.81 \times 0.9 = 8.83$	5.89		0.76



$v = ki = k\frac{\Delta h}{L} = 0.5 \times \frac{\frac{23.4}{9.81}}{2.4-0.6} = 0.66$ cm/sec

$v = \frac{ki}{n} = \frac{0.5 \times \frac{\frac{23.4}{9.81} - 0.90}{2.4-0.6}}{0.33} = 1.24$ cm/sec

Fig. 17.10 Example of horizontal flow.

Length	0	0.6	1.2	1.8	2.4	3.0, 3.6
$h_t = h_p = 0.9 + (\frac{p_a}{\gamma_w} - 0.9)$						
h_t	$\frac{23.4}{9.81} = 2.38$	$\frac{23.4}{9.81} = 2.38$	$\frac{23.4}{9.81} - \frac{\frac{23.4}{9.81}}{2.4-0.6} \times (1.2-0.6) = 1.59$	$2.38 - \frac{2.38}{2.4-0.6} \times (1.8-0.6) = 0.80$	$2.38 - \frac{2.38}{2.4-0.6} \times (2.4-0.6) = 0$	0
h_e	0.90	0.90	0.90	0.90	0.90	0.90
$h_p = h_t - h_e$	1.48	1.48	0.69	-0.10	-0.90	-0.90

$q_1 = q_2$; $h = \Delta h_1 + \Delta h_2$

$k_1 \cdot i_1 A_1 = k_2 i_2 \cdot A_2$; $\Delta h_1 = h_1 - h_2$; $h_1 = h$

$\Delta h_2 = h_2 - 0 = h_2$

$k_1 \cdot \frac{\Delta h_1}{L_1} \cdot A_1 = k_2 \frac{\Delta h_2}{L_2} \cdot A_2$

$\Delta h_1 = \frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot \frac{A_2}{A_1} \cdot \Delta h_2$

$= \frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot \frac{A_2}{A_1} \cdot (h - \Delta h_1)$

$\Delta h_1 = \frac{\frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot \frac{A_2}{A_1} \cdot h}{1 + \frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot \frac{A_2}{A_1}}$

$\Delta h_2 = h - \Delta h_1$

$= h - \frac{\frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot \frac{A_2}{A_1} \cdot h}{1 + \frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot \frac{A_2}{A_1}}$

$= \frac{h}{1 + \frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot \frac{A_2}{A_1}}$

$\Delta h_1 = \frac{\frac{0.5}{1} \times \frac{1.2}{0.6} \times \frac{0.186}{0.37} \times 3.6}{1 + \frac{0.5}{1} \times \frac{1.2}{0.6} \times \frac{0.186}{0.37}} = 1.20$

$\Delta h_2 = \frac{3.6}{1 + \frac{0.5}{1} \times \frac{1.2}{0.6} \times \frac{0.186}{0.37}} = 2.40$

$0 \le x \le L_1$ $\quad h_t = (h_e + h_p) - \frac{h_1 - h_2}{L_1} \cdot x$

$= (h + 0) - \frac{\Delta h_1}{L_1} \cdot x$

$= h - \frac{\Delta h_1}{L_1} \cdot x$

$x = 0$ $\quad h_t = h - \frac{\Delta h_1}{L_1} \cdot 0 = h$

$x = L_1$ $\quad h_t = h - \frac{\Delta h_1}{\cancel{L_1}} \cdot \cancel{L_1} = h - \Delta h_1 = h_2$

$\frac{h_2 - 0}{L - L_1 \rightarrow L_2} = \frac{h_2 - x}{x - L_1} \dashrightarrow h_2 - x = \frac{h_2}{L_2} \cdot (x - L_1) = \frac{\Delta h_2}{L_2} \cdot (x - L_1)$

$L_1 \le x \le L_1 + L_2$ $\quad h_t = (h - \Delta h_1) - \frac{\frac{\Delta h_2}{L_2} \cdot (x - L_1)}{\cancel{x - L_1}} \cdot \cancel{(x - L_1)}$

$= \Delta h_2 - \frac{\Delta h_2}{L_2}(x - L_1) = \Delta h_2 - \frac{\Delta h_2}{L_2} x + \frac{\Delta h_2}{L_2} \cdot L_1$

$= \Delta h_2 \left(1 + \frac{L_1}{L_2}\right) - \frac{\Delta h_2}{L_2} x$

$x = L_1$ $\quad h_t = \Delta h_2 \left(1 + \frac{L_1}{L_2}\right) - \frac{\Delta h_2}{L_2} \cdot L_1$

$= \Delta h_2 \left(1 + \frac{L_1}{L_2} - \frac{L_1}{L_2}\right)$

$= \Delta h_2 = h_2$

$x = L_1 + L_2$ $\quad h_t = \Delta h_2 \left(1 + \frac{L_1}{L_2}\right) - \frac{\Delta h_2}{L_2} \cdot (L_1 + L_2)$

$= \Delta h_2 \left(1 + \frac{L_1}{L_2} - \frac{L_1}{L_2} - \frac{L_2}{L_2}\right)$

$= 0$

$A_1 = 0.37\ m^2$

$n_1 = 1/2$

$k_1 = 1$ cm/sec

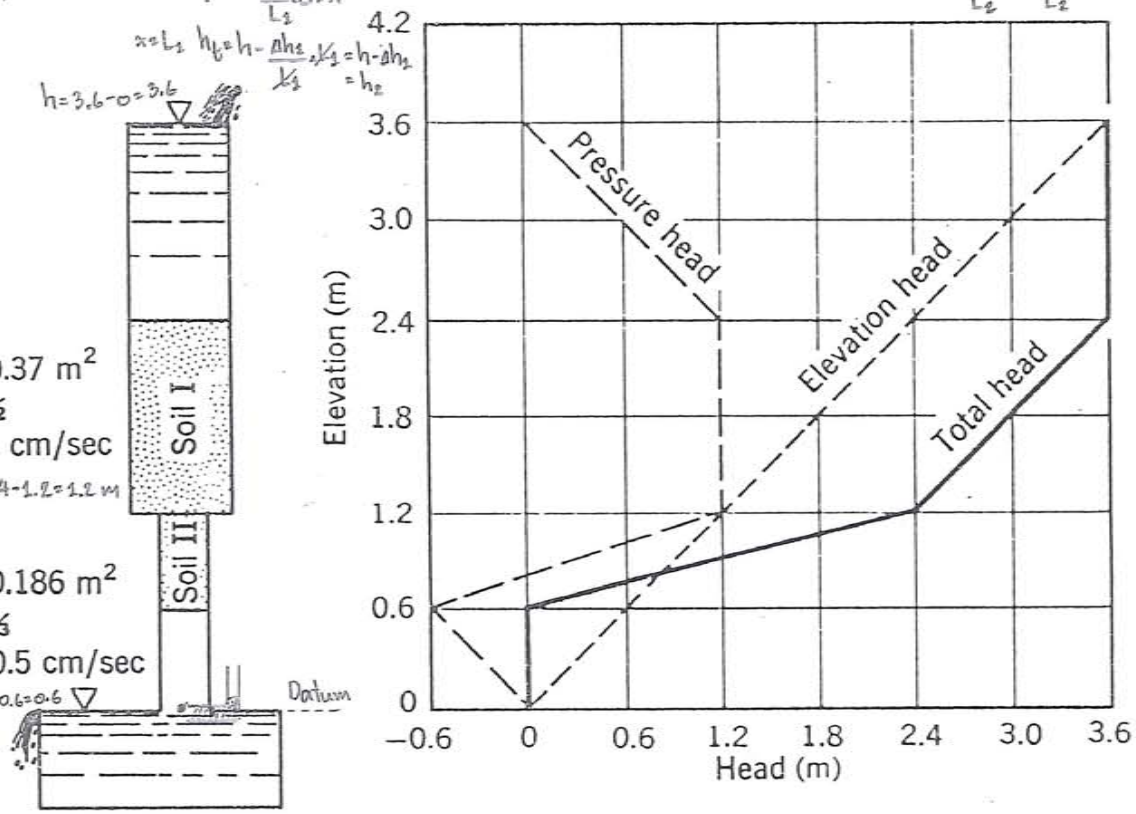
$L_1 = 2.4 - 1.2 = 1.2$ m

$A_2 = 0.186\ m^2$

$n_2 = 1/3$

$k_2 = 0.5$ cm/sec

$L_2 = 1.2 - 0.6 = 0.6$



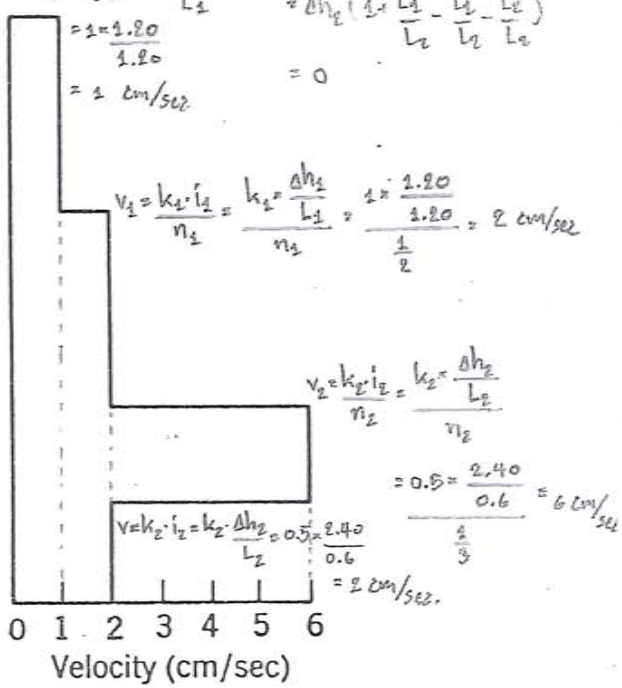
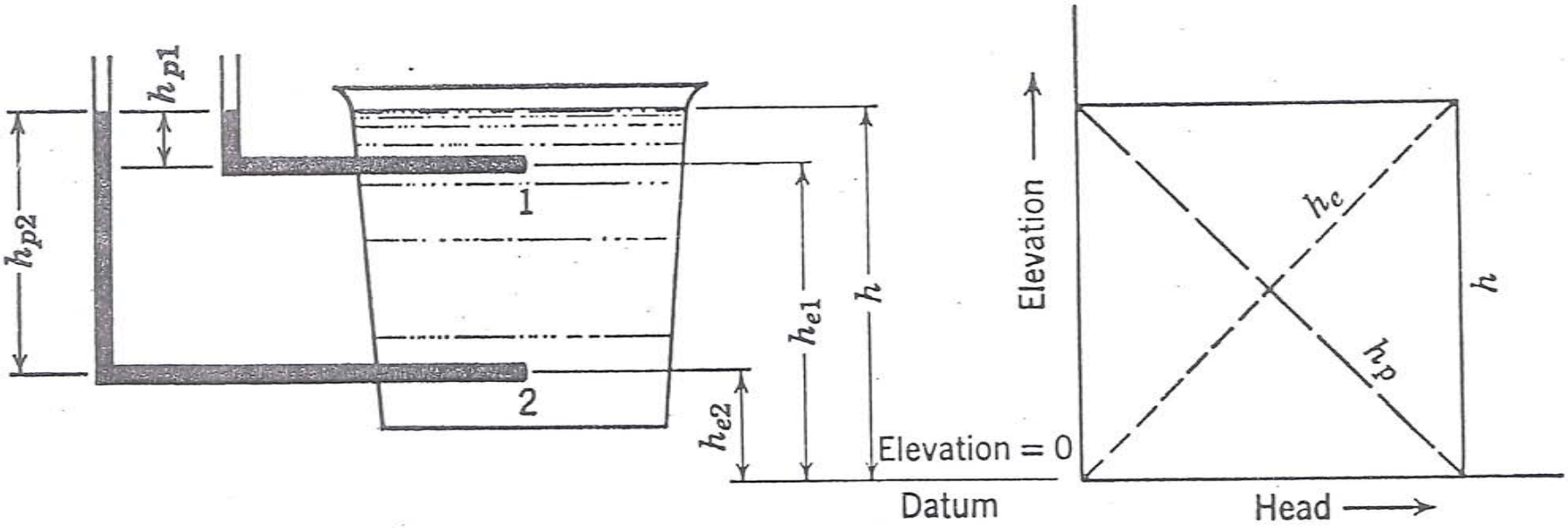


Fig. 17.11 Example of flow through two soils in series.

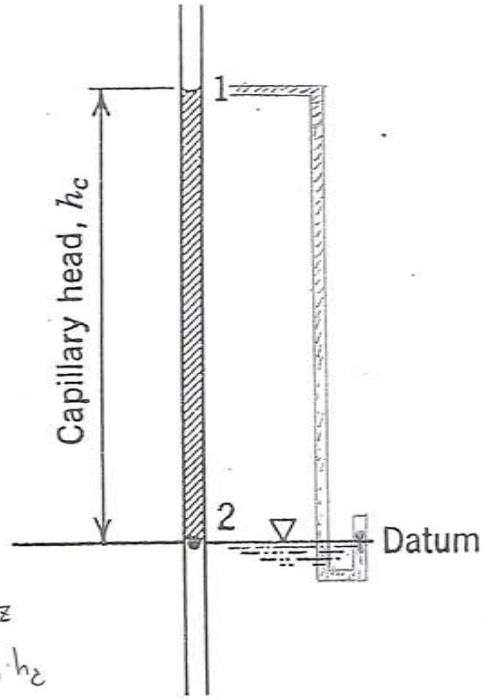
Elev.	h_t	h_e	$h_p = h_t - h_e$
3.6	$h_e + h_p = 3.6 + 0 = 3.6$	3.6	0
2.4	$h_e + h_p = 2.4 + (3.6 - 2.4) = 3.6$	2.4	1.2
1.8	$h - \frac{\Delta h_1}{L_1} \cdot x = 3.6 - \frac{1.20}{1.2}(2.4 - 1.8) = 3.0$	1.8	1.2
1.2	$h - \Delta h_1 = h_2 = \Delta h_2 = 3.6 - 1.20 = 2.4$	1.2	1.2
0.6	$h_2 - \Delta h_2 = 2.4 - 2.4 = 0$	0.6	-0.6
0	$h_e + h_p = 0 + 0 = 0$	0	0

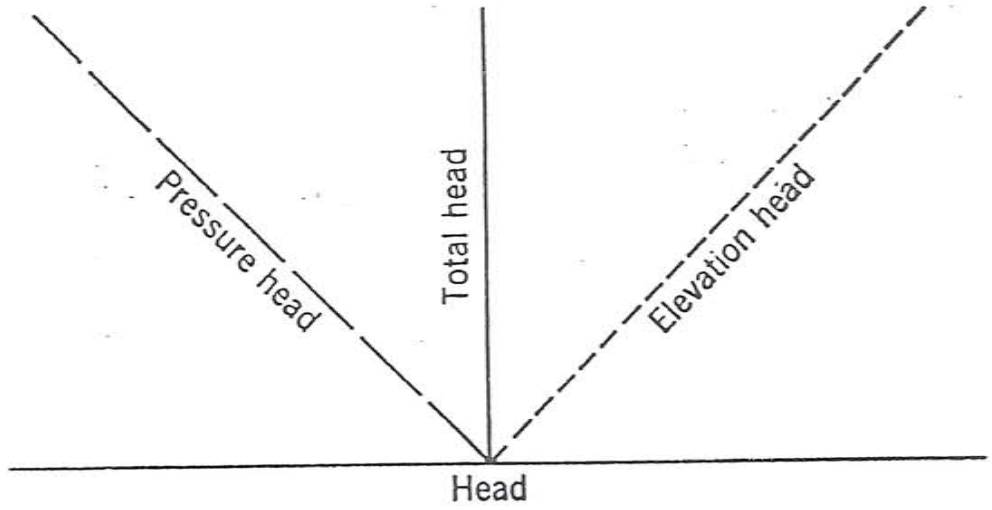


Point	Elevation Head	Pressure Head	Total Head
1	h_{e1}	h_{p1}	$h_{e1} + h_{p1} = h$
2	h_{e2}	h_{p2}	$h_{e2} + h_{p2} = h$

(a)

total head ทุกจุด ตรงที่
ไม่มี flow เกิดขึ้น

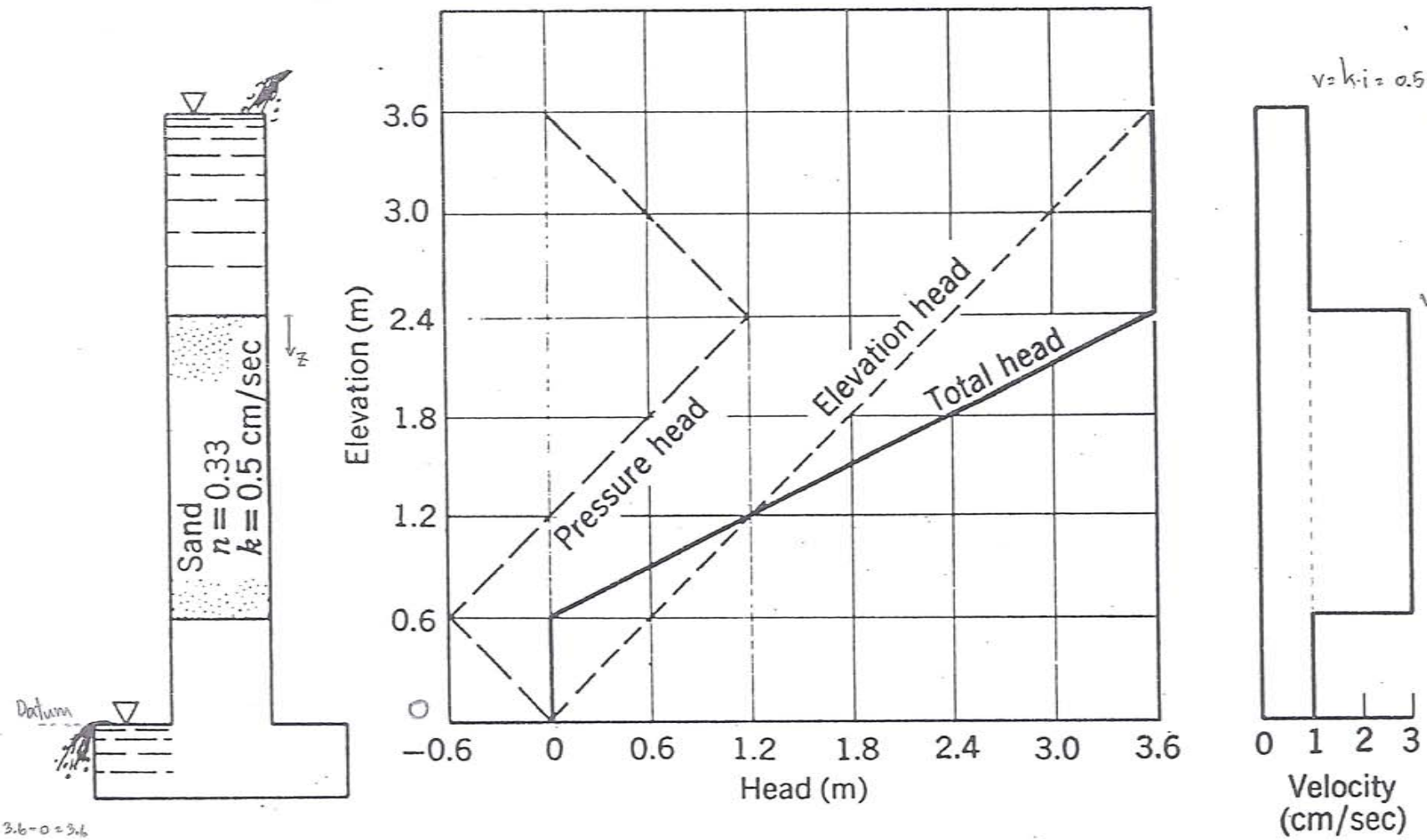




Point	Elevation Head	Pressure Head	Total Head
1	h_c	$-h_c$	$h_c - h_c = 0$
2	0	0	0

(*b*)

Fig. 17.4 Heads in static water. (*a*) In container. (*b*) In capillary tube.



$v = k \cdot i = 0.5 \times \frac{3.6}{2.4-0.6} = 1$ cm/sec

$v = \frac{k \cdot i}{n} = \frac{0.5 \times \frac{3.6}{2.4-0.6}}{0.33} = 3$ cm/sec

$h = 3.6 - 0 = 3.6$

$h_t = 3.6 - \frac{3.6}{2.4-0.6} \cdot z$

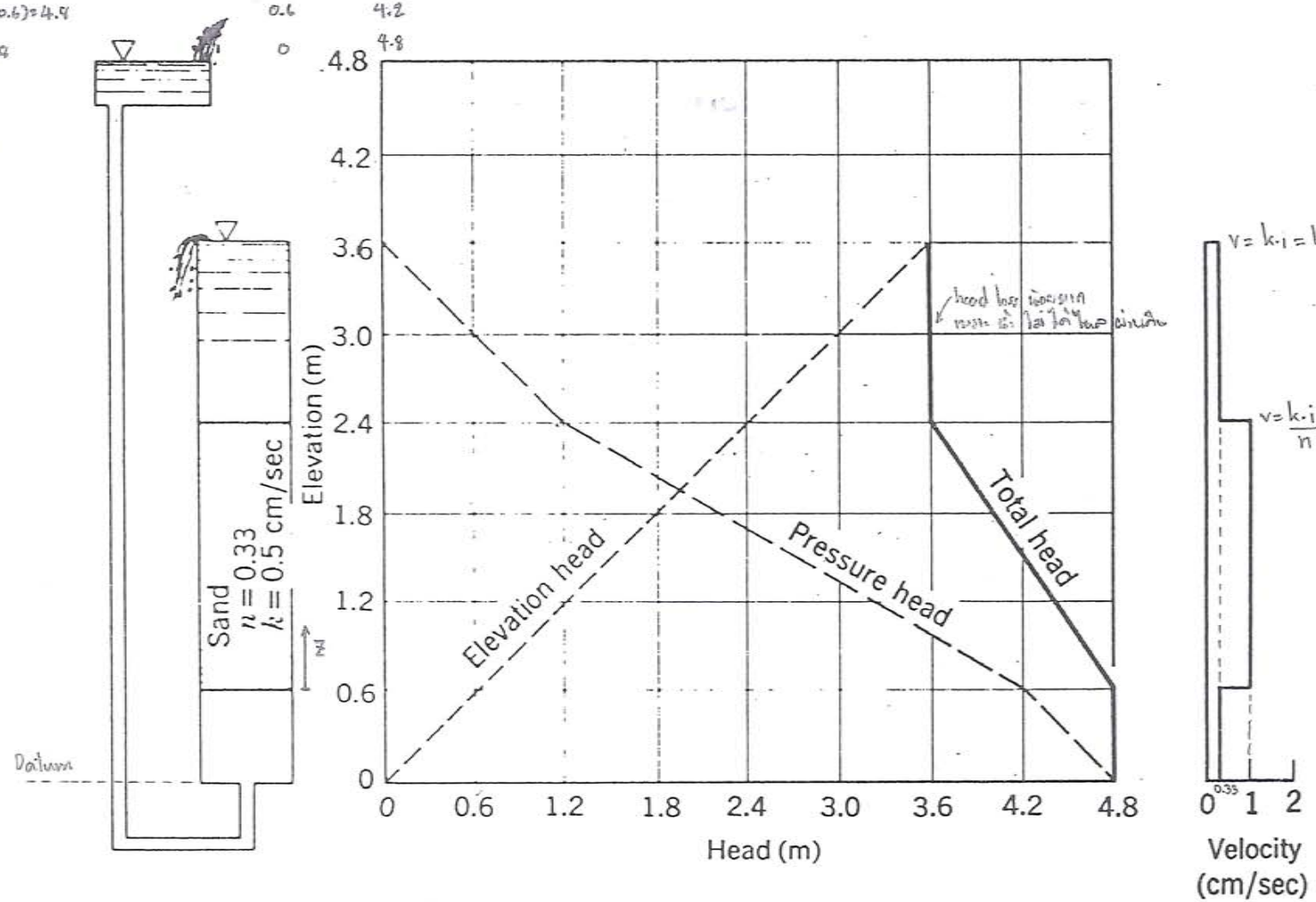
Elev	h_t	h_e	$h_p = h_t - h_e$
3.6	$h_e + h_p = 3.6 + 0 = 3.6$	3.6	0
2.4	$h_e + h_p = 2.4 + (3.6 - 2.4) = 3.6$	2.4	1.2
1.8	$3.6 - \frac{3.6}{2.4-0.6} \times (2.4 - 1.8) = 2.4$	1.8	0.6
1.2	$3.6 - \frac{3.6}{2.4-0.6} \times (2.4 - 1.2) = 1.2$	1.2	0
0.6	$3.6 - \frac{3.6}{2.4-0.6} \times (2.4 - 0.6) = 0$	0.6	-0.6
0	0	0	0

Fig. 17.8 Example of downward flow.

Elev.	h_t	h_e	$h_p = h_t - h_e$
3.6	3.6	3.6	0
2.4	$2.4+(4.8-2.4)-\frac{1.2}{2.4-0.6}\times(2.4-0.6)=3.6$	2.4	1.2
1.8	$1.8+(4.8-1.8)-\frac{1.2}{2.4-0.6}\times(1.8-0.6)=4.0$	1.8	2.2
1.2	$1.2+(4.8-1.2)-\frac{1.2}{2.4-0.6}\times(1.2-0.6)=4.4$	1.2	3.2
0.6	$h_e+h_p=0.6+(4.8-0.6)=4.8$	0.6	4.2
0	$h_e+h_p=0+4.8=4.8$	0	4.8

$$h_t=(h_e+h_p)-\frac{h}{2.4-0.6}\times z$$

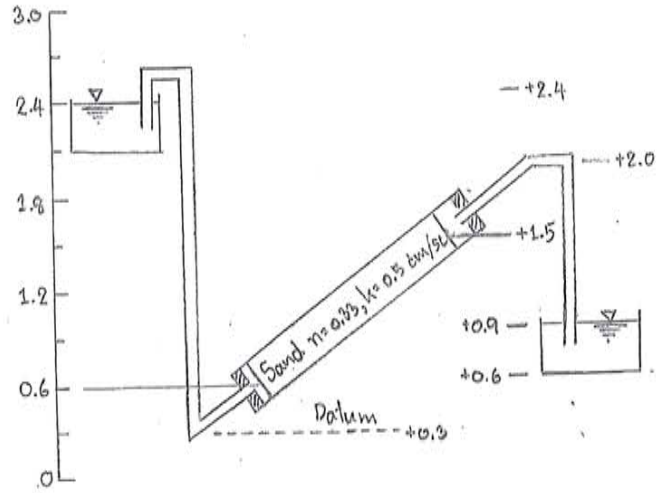
$$h=4.8-3.6=1.2$$



$$v=k\cdot i=k\times\frac{\Delta h}{L}=0.5\times\frac{4.8-3.6}{2.4-0.6}=0.5\times\frac{1.2}{1.8}$$
$$=0.33\ \text{cm/sec}$$

$$v=\frac{k\cdot i}{n}=\frac{k\times\frac{\Delta h}{L}}{n}=\frac{0.5\times\frac{1.2}{1.8}}{0.33}=1\ \text{cm/sec}$$

Fig. 17.9 Example of upward flow.

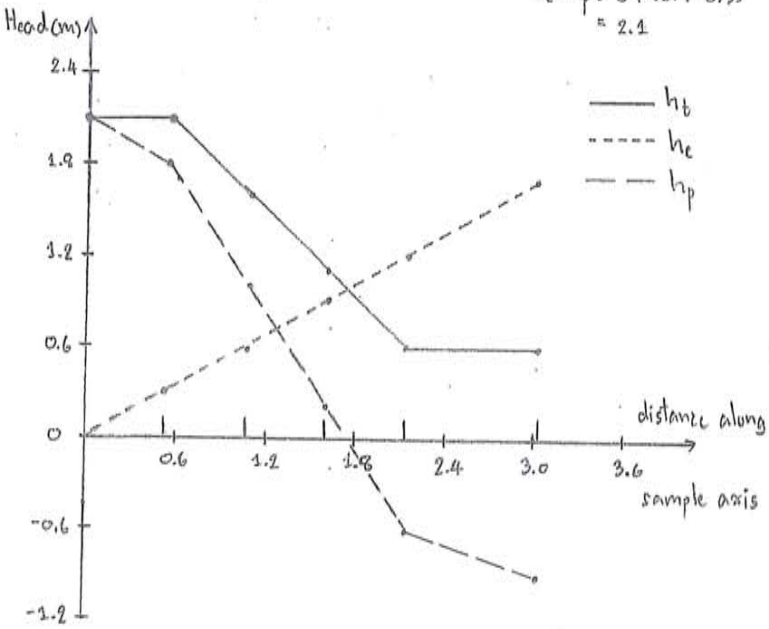


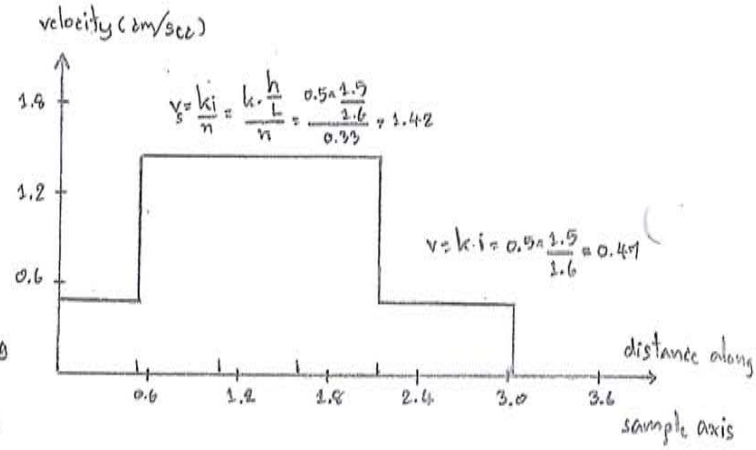
$L = 1.6$ m

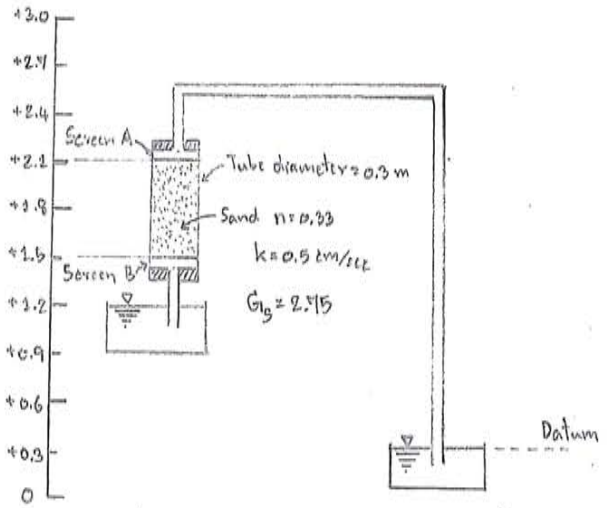
$\theta = \sin^{-1}\left(\frac{1.5-0.6}{1.6}\right) = 34.23^\circ$

$h = 2.4 - 0.9 = 1.5$

Elev. (m)	distance along sample axis (m)	h_t	h_e	$h_p = h_t - h_e$
2.0	$\frac{2.0-0.3}{\sin 34.23} = 3.02$	$h_e + h_p = (0.9-0.3)+0 = 0.6$	$2.00-0.30 = 1.7$	-0.9
1.5	$\frac{1.5-0.3}{\sin 34.23} = 2.13$	$h_t = \frac{h}{L} \times L = h_t = h = 2.1-1.5 = 0.6$	$1.50-0.30 = 1.2$	-0.6
1.2	$\frac{1.2-0.3}{\sin 34.23} = 1.60$	$h_t = \frac{h}{L} \times x = 2.1 - \frac{1.5}{1.60} \times (1.60-0.53) = 1.1$	$1.20-0.30 = 0.9$	0.2
0.9	$\frac{0.9-0.3}{\sin 34.23} = 1.07$	$h_t = \frac{h}{L} \times x = 2.1 - \frac{1.5}{1.60} \times (1.07-0.53) = 1.6$	$0.90-0.30 = 0.6$	1.0
0.6	$\frac{0.6-0.3}{\sin 34.23} = 0.53$	$h_e + h_p = (0.6-0.3)+(2.4-0.6) = 2.1$	$0.60-0.30 = 0.3$	1.8
0.3	0	$h_e + h_p = 0 + (2.4-0.3) = 2.1$	$0.30-0.30 = 0$	2.1







$h = 1.2 - 0.3 = 0.9$ m

$L = 2.1 - 1.5 = 0.6$ m

$A = \frac{\pi}{4} \times 0.3^2 = 0.0707$ m^2

$\gamma_t = [G_s - n(G_s - 1)]\gamma_w = [2.75 - 0.33(2.75 - 1)] \times 9.81$

$= 21.31$ kN/m^3

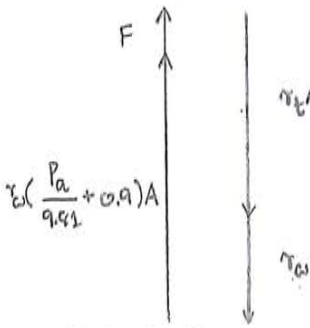
- Vertical force exerted by soil on screen B

Elev.	h_t	h_e	$h_p = h_t - h_e$	h_{ps}	$u = \gamma_w h_p$	$u_s = \gamma_w h_{ps}$	$\Delta u = u - u_s$
2.1	$h_t = \frac{h + \frac{P_a}{\gamma_w}}{L} \times L = \left(\frac{P_a}{9.81} + 0.9\right) - \left(0.9 + \frac{P_a}{9.81}\right) = 0$	1.8	-1.8	$-(2.1-0.3) = -1.8$	-17.66	-17.66	0
1.5	$h_e + h_p = (1.5-0.3) + \left(\frac{P_a}{9.81} - (1.5-1.2)\right) = \frac{P_a}{9.81} + 0.9$	1.2	$\frac{P_a}{9.81} - 0.3$	$-(1.5-0.3) = -1.2$	$P_a - 2.94$	-11.77	$P_a + 8.83$
1.2	$h_e + h_p = (1.2-0.3) + \frac{P_a}{9.81} = \frac{P_a}{9.81} + 0.9$	0.9	$\frac{P_a}{9.81}$	$-(1.2-0.3) = -0.9$	P_a	-8.83	$P_a + 8.83$
0.3	$h_e + h_p = 0 + 0 = 0$	0	0	0	0	0	0

$\Delta u = P_a + 8.83 = 0$; $F = 0$ for No flowing

$P_a = -8.83$ kN/m^2

ที่ Elev. 1.5



$$F + \gamma_w\left(\frac{P_a}{9.81} + 0.9\right)A = \gamma_t AL + \gamma_w h_{ps} A$$

$$F + 9.81\left(\frac{-8.83}{9.81} + 0.9\right) \times 0.07 = 21.31 \times 0.07 \times 0.6 + 9.81 \times (-1.2) \times 0.07$$

$$F = 0.0712 \text{ kN} \uparrow$$

- Vertical force exerted by soil on screen A

Elev.	h_t	h_e	$h_p = h_t - h_e$	h_{ps}	$u = \gamma_w h_p$	$u_s = \gamma_w h_{ps}$	$\Delta u = u - u_s$
0.3	$h_e + h_p = 0 + \frac{P_a}{\gamma_w} = \frac{P_a}{9.81}$	0	$\frac{P_a}{9.81}$	$1.2 - 0.3 = 0.9$	P_a	8.83	$P_a - 8.83$
2.1	$h_e + h_p = (2.1-0.3) + \left(\frac{P_a}{9.81} - (2.1-0.3)\right) = \frac{P_a}{9.81}$	1.8	$\frac{P_a}{9.81} - 1.8$	$-(2.1-1.2) = -0.9$	$P_a - 17.66$	-8.83	$P_a - 8.83$
1.5	$h_t = \frac{\frac{P_a}{\gamma_w} - h}{L} \times L = \frac{P_a}{9.81} - \left(\frac{P_a}{9.81} - 0.9\right) = 0.9$	1.2	-0.3	$-(1.5-1.2) = -0.3$	-2.94	-2.94	0
1.2	$h_e + h_p = (1.2-0.3) + 0 = 0.9$	0.9	0	0	0	0	0

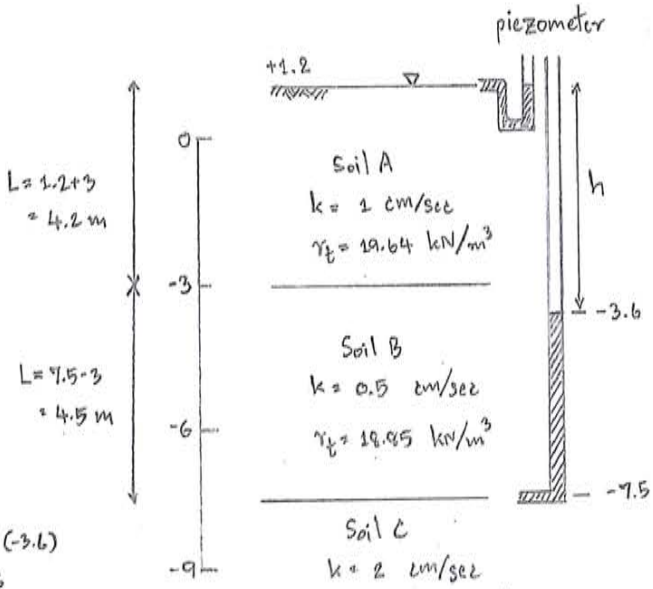
$\Delta u = P_a - 8.83 = 0$; $F = 0$ for No flowing

$P_a = 8.83$ kN/m^2

$F + \gamma_t AL + \gamma_w h_{ps} A + \frac{P_a}{9.81} \times \gamma_w A = 0$

$F + 21.31 \times 0.07 \times 0.6 + 9.81 \times (-0.9) \times 0.07 + \frac{8.83}{9.81} \times 9.81 \times 0.07 = 0$

$F = -0.895$ kN



$\gamma_t = [G_s - n(G_s - 1)]\gamma_w$ (eq. 1.18, "Adv. soil mech", Das)

$n = \dfrac{G_s - \frac{\gamma_t}{\gamma_w}}{G_s - 1}$

soil A $n_1 = \dfrac{2.75 - \frac{19.64}{9.81}}{2.75 - 1} = 0.43$

soil B $n_2 = \dfrac{2.75 - \frac{18.85}{9.81}}{2.75 - 1} = 0.47$

$q_1 = q_2$

$k_1 i_1 A_1 = k_2 i_2 A_2$

consider 1 unit of area

$k_1 i_1 = k_2 i_2$

$k_1 \dfrac{\Delta h_1}{L_1} = k_2 \dfrac{\Delta h_2}{L_2}$

$\Delta h_1 = \dfrac{k_2}{k_1} \cdot \dfrac{L_1}{L_2} \Delta h_2$

$= \dfrac{k_2}{k_1} \cdot \dfrac{L_1}{L_2} \cdot (h - \Delta h_1)$

$\Delta h_1 = \dfrac{\frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot h}{1 + \frac{k_2}{k_1} \cdot \frac{L_1}{L_2}}$

$\Delta h_2 = h - \Delta h_1$

$\Delta h_2 = h - \dfrac{\frac{k_2}{k_1} \cdot \frac{L_1}{L_2} \cdot h}{1 + \frac{k_2}{k_1} \cdot \frac{L_1}{L_2}}$

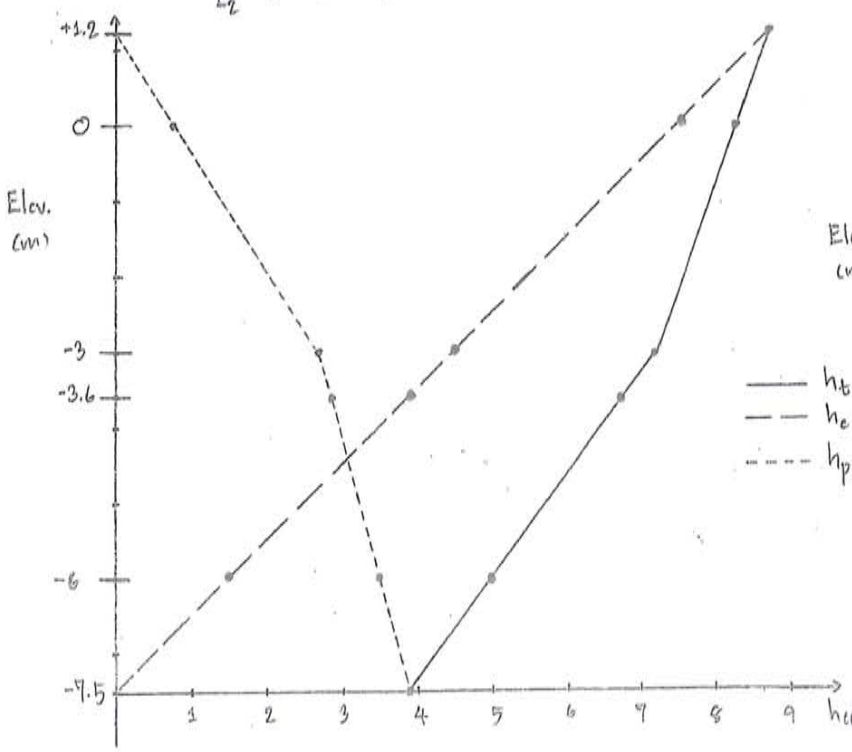
$\Delta h_2 = \dfrac{h}{1 + \frac{k_2}{k_1} \cdot \frac{L_1}{L_2}}$

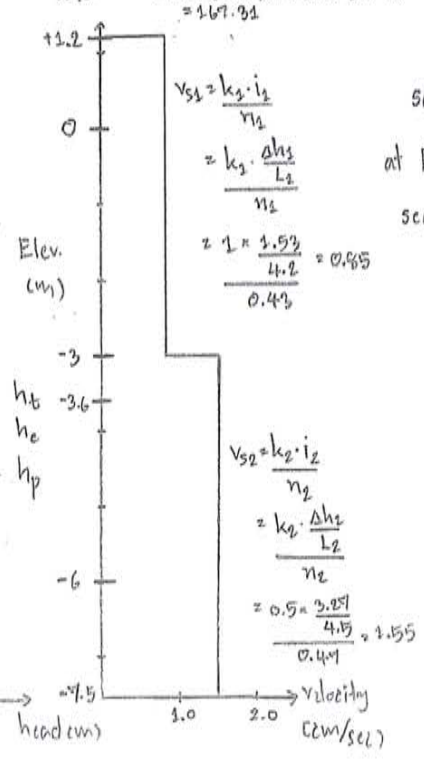
$h = 1.2 - (-3.6)$
$= 4.8$

$\Delta h_1 = \dfrac{\frac{0.5}{1} \times \frac{4.2}{4.5} \times 4.8}{1 + \frac{0.5}{1} \times \frac{4.2}{4.5}} = 1.53$

$\Delta h_2 = \dfrac{4.8}{1 + \frac{0.5}{1} \times \frac{4.2}{4.5}} = 3.27$

Elev. (m)	h_t (m)	h_e (m)	$h_p = h_t - h_e$ (m)	σ_v (kN/m²)	$u = \gamma_w h_p$ (kN/m²)	$\sigma_v' = \sigma_v - u$ (kN/m²)
+1.2	$h_t = h_p = (7.5+1.2)+0$ $= 8.7 = h_{t1}$	7.5+1.2 = 8.7	0	0	0	0
0	$h_{t1} - \frac{\Delta h_1}{L_1} \times z = 8.7 - \frac{1.53}{4.2} \times (1.2-0) = 8.26$	7.5	0.76	19.64×(1.2−0) = 23.57	7.46	16.11
-3	$h_{t1} - \frac{\Delta h_1}{L_1} \times L_1 = h_{t1} - \Delta h_1 = 8.7 - 1.53 = 7.17 = h_{t2}$	7.5−3 = 4.5	2.67	19.64×(1.2+3) = 82.49	26.19	56.30
-3.6	$h_{t2} - \frac{\Delta h_2}{L_2} \times z = 7.17 - \frac{3.27}{4.5} \times (3.6-3) = 6.73$	7.5−3.6 = 3.9	2.83	19.64×(1.2+3)+18.85×(3.6−3) = 93.80	27.76	66.04
-6	$h_{t2} - \frac{\Delta h_2}{L_2} \times z = 7.17 - \frac{3.27}{4.5} \times (6-3) = 4.99$	7.5−6 = 1.5	3.49	19.64×(1.2+3)+18.85×(6−3) = 139.64	34.24	104.80
-7.5	$h_{t2} - \frac{\Delta h_2}{L_2} \times L_2 = h_{t2} - \Delta h_2 = 7.17 - 3.27 = 3.90$	0	3.90	19.64×(1.2+3)+18.85×(7.5−3) = 167.31	38.26	129.05





$v_{s1} = \dfrac{k_1 \cdot i_1}{n_1} = \dfrac{k_1 \cdot \frac{\Delta h_1}{L_1}}{n_1} = \dfrac{1 \times \frac{1.53}{4.2}}{0.43} = 0.85$

$v_{s2} = \dfrac{k_2 \cdot i_2}{n_2} = \dfrac{k_2 \cdot \frac{\Delta h_2}{L_2}}{n_2} = \dfrac{0.5 \times \frac{3.27}{4.5}}{0.47} = 1.55$

seepage force = $i \cdot \gamma_w \cdot V$

at Elev. −4.5 m

seepage force = $\dfrac{\Delta h_2}{L_2} \times \gamma_w \cdot V$

$= \dfrac{3.27}{4.5} \times 9.81 \times (0.3 \times 0.3 \times 0.3)$

$= 0.192$ kN

▶ Example 18.2

Given: Flow net in Fig. E18.2

Find: Pressure heads at points A to H; quantity of seepage; gradient in X

Solution: The pressure heads are shown in Fig. E18.2.

Seepage:

$\frac{b}{l} = \frac{1.2}{0.9} \approx \frac{1}{0.6}$

$$n_f = 4 \qquad n_d = 12.6 \qquad k = 0.05 \text{ cm/sec} \qquad \$ = \frac{n_f}{n_d} = 0.317$$

$$\frac{Q}{L} = kH\$ = 0.05 \times 10^{-2} \text{ m/sec} \times 7.8 \text{ m} \times 0.317 = 0.00124 \text{ (m}^3\text{/sec)(m)}$$

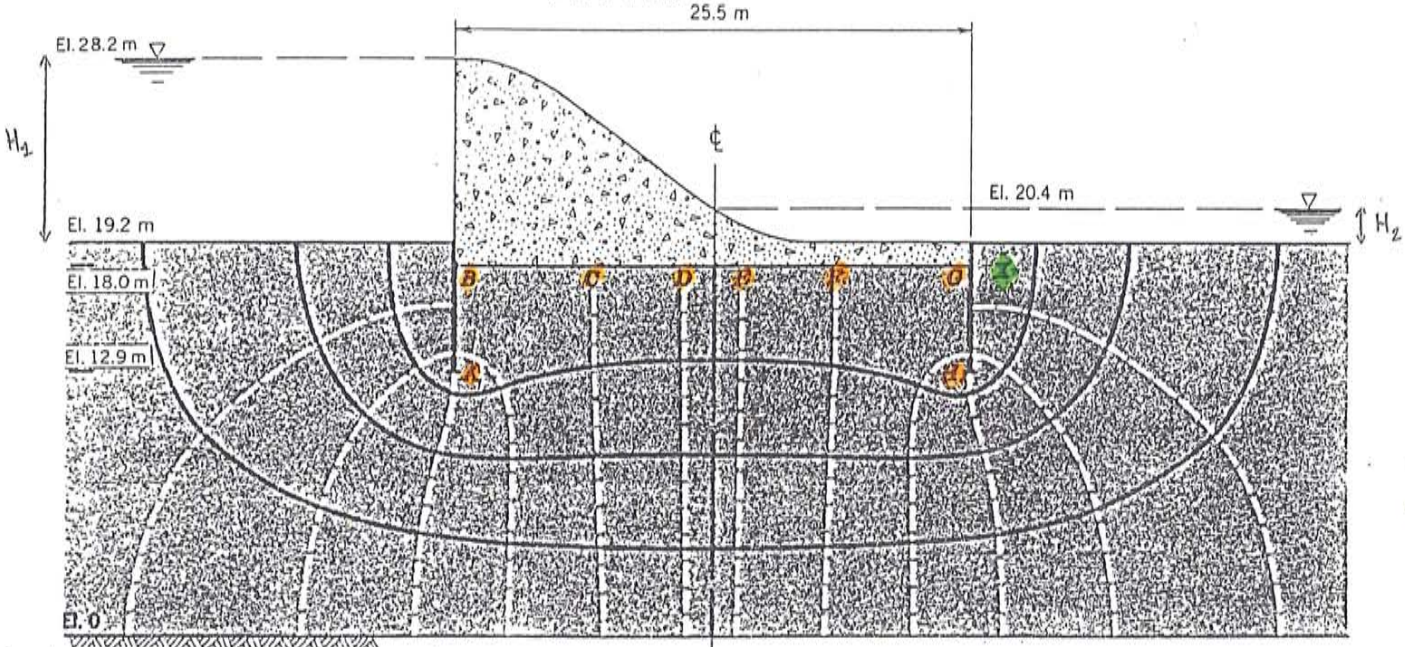
$28.2 - 20.4$

Gradient in X:

$\frac{28.2-20.4}{12.6} = \frac{\Delta H}{n_d}$

$$i_X = \frac{\Delta h}{l} = \frac{0.62}{3.3} = 0.19$$

ระยะ x ที่ท้ายเขื่อน



$h_p = h_t - h_e = (H_1 - n_d \times \frac{H_1 - H_2}{n_d}) - h_e$

$H_1 - H_2 = 28.2 - 20.4 = 7.8$

จุด A $h_p = (28.2 - 3.0 - \frac{7.8}{12.6}) - 12.9$
$= 13.4$

จุด B $h_p = (28.2 - 4.5 - \frac{7.8}{12.6}) - 18.0$
$= 7.4$

จุด C $h_p = (28.2 - 5.0 - \frac{7.8}{12.6}) - 18.0$
$= 7.1$

จุด D $h_p = (28.2 - 6.0 - \frac{7.8}{12.6}) - 18.0$
$= 6.5$

$6.0 + 0.6$

จุด E $h_p = (28.2 - 6.6 - \frac{7.8}{12.6}) - 18.0$
$= 6.1$

$6.6 + 1.0$

จุด F $h_p = (28.2 - 7.6 - \frac{7.8}{12.6}) - 18.0$
$= 5.5$

$7.6 + 0.5$

จุด G $h_p = (28.2 - 8.1 - \frac{7.8}{12.6}) - 18.0$
$= 5.2$

$7.6 + 2.0$

จุด H $h_p = (28.2 - 9.6 - \frac{7.8}{12.6}) - 12.9$
$= 9.4$

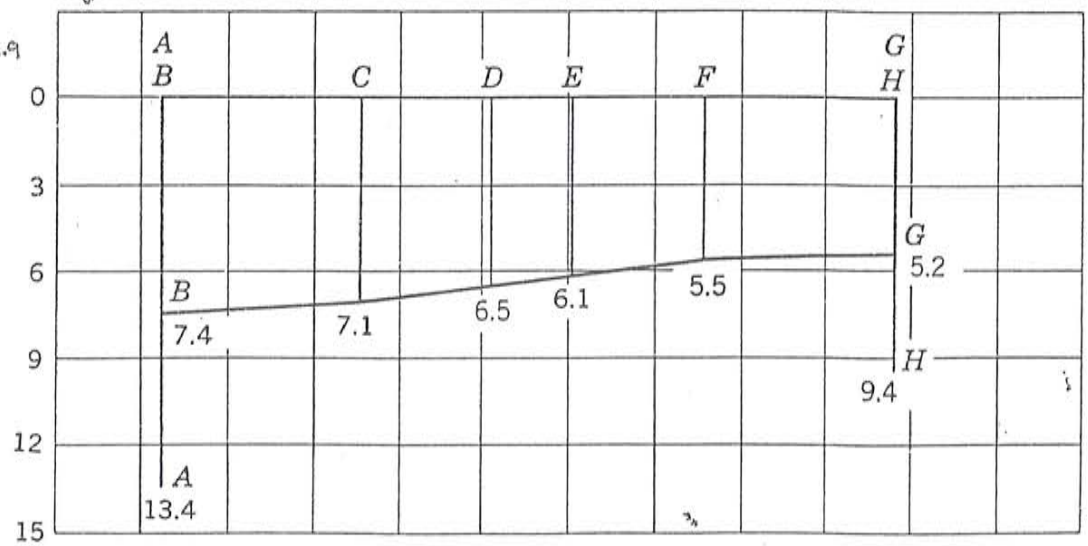
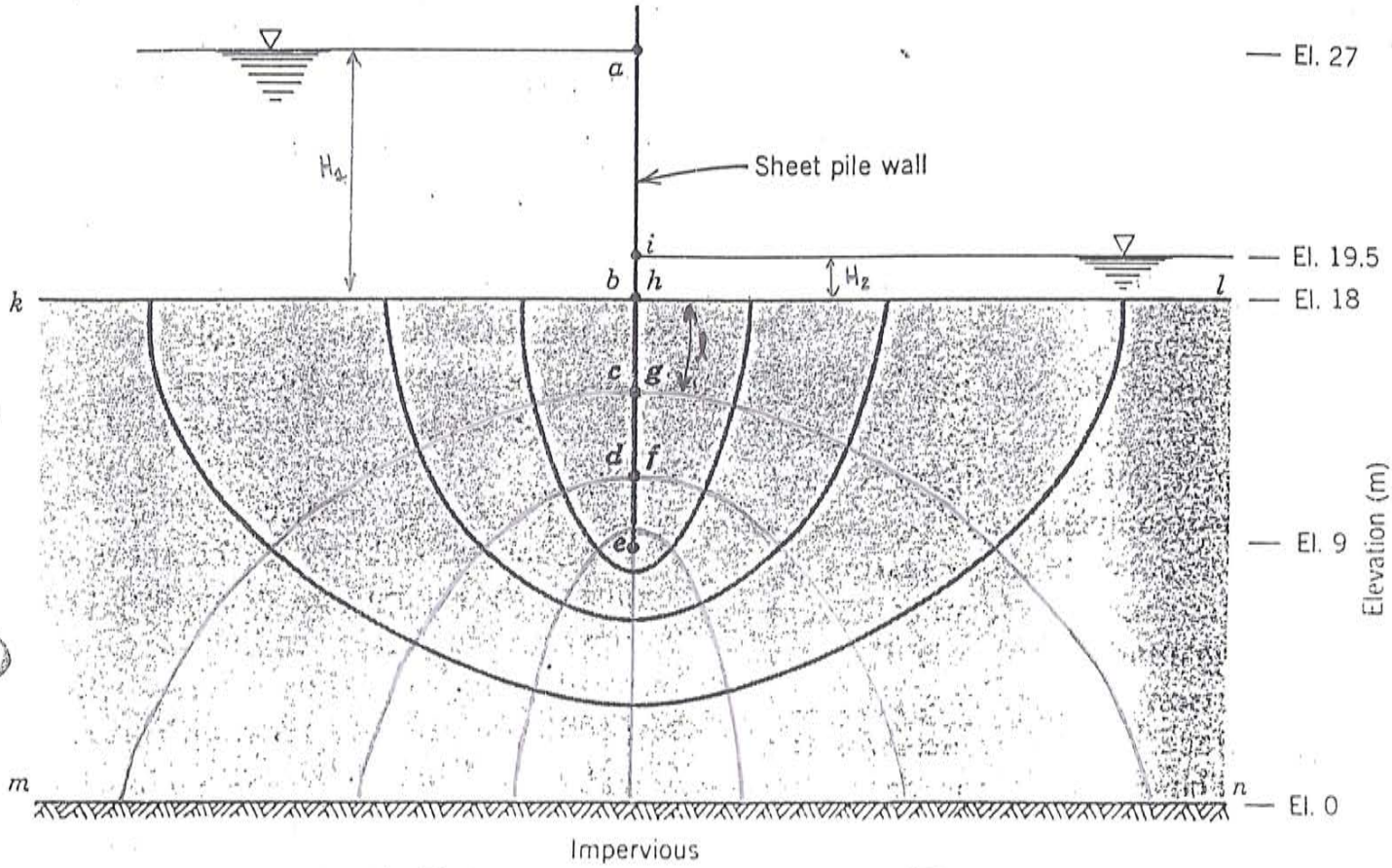


Fig. E18.2 Flow under dam.



▶ Example 18.1

Given. Flow net in Fig. E18.1.

Find. Pore pressures at points a to i; quantity of seepage; exit gradient.

Solution.

h_e $\quad h_{t_i} = h_{t@\,n_d\,b} - n_{d_i} \times \frac{H_1 - H_2}{n_d}$ $\quad h_p = h_t - h_e$ $\quad u = \gamma_w h_p$; $\gamma_w = 9.81$ kN/m³

Point	Elevation Head (m)	Total Head (m)	Pressure Head (m)	Water Pressure (kN/m²)
a	27.0	$h_e + h_p = 27.0 + 0 = 27.0$	0	0
b	18.0	$h_e + h_p = 18.0 + (27 - 18) = 27.0$	9.00	88.3
c	14.7	$27.0 - 1.0 \times \frac{27 - 19.5}{8.0} = 26.06$	11.36	111.4
d	11.7	$27.0 - 2.0 \times \frac{27 - 19.5}{8.0} = 25.13$	13.43	131.7
e	9.0	$27.0 - 4.0 \times \frac{27 - 19.5}{8.0} = 23.25$	14.25	139.8
f	11.7	$27.0 - 6.0 \times \frac{27 - 19.5}{8.0} = 21.38$	9.68	95.0
g	14.7	$27.0 - 7.0 \times \frac{27 - 19.5}{8.0} = 20.44$	5.74	56.3
h	18.0	$27.0 - 8.0 \times \frac{27 - 19.5}{8.0} = 19.5$	1.50	14.7
i	19.5	$h_e + h_p = 19.5 + 0 = 19.5$	0	0

Seepage under wall:

$n_f = 4$

$$\frac{Q}{L} = kH\frac{n_f}{n_d} = 5 \times 10^{-9}\ \text{m/sec} \times 7.5 \times \frac{4}{8}$$

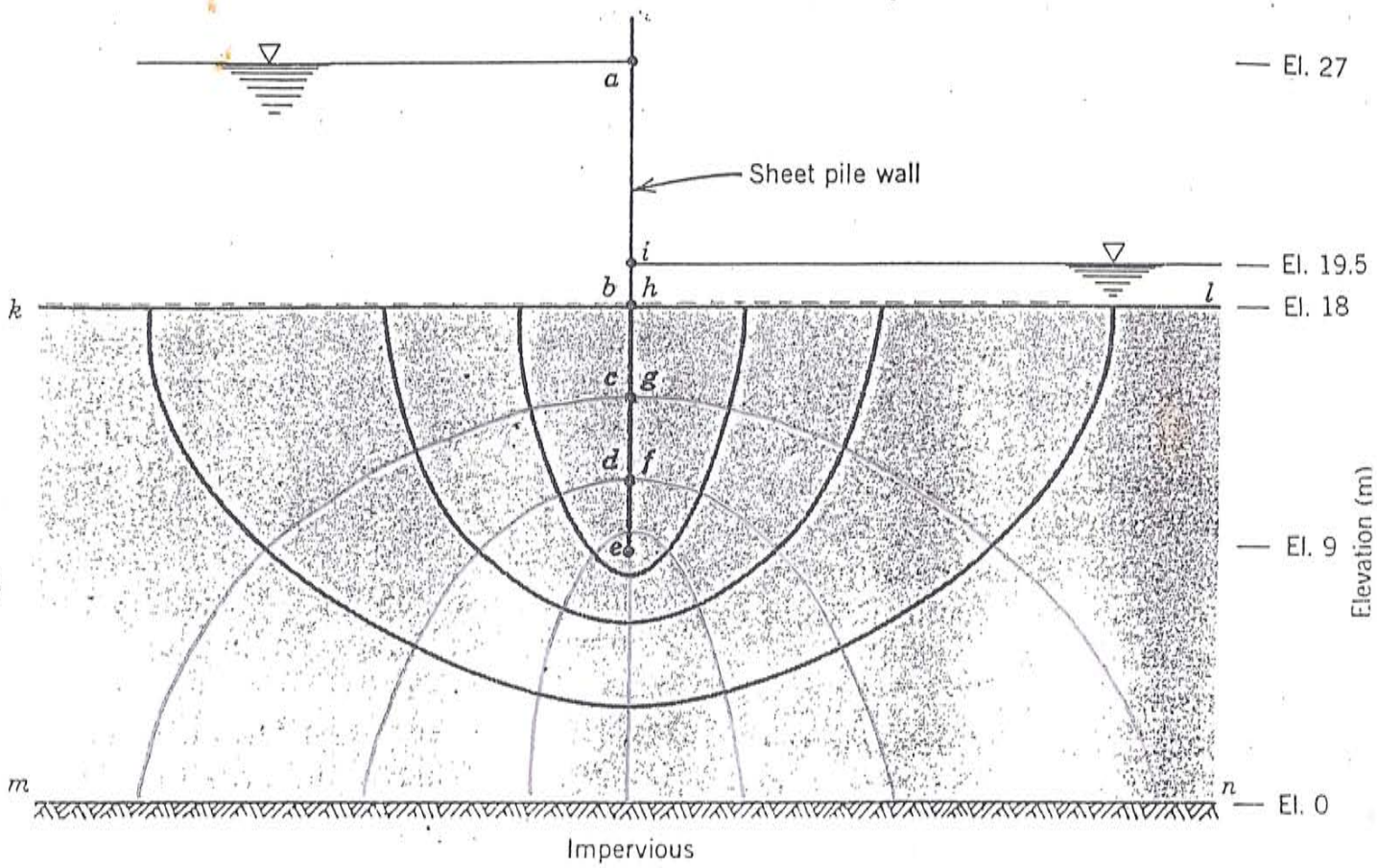
$n_d = 8$

$$= 18.75 \times 10^{-9}\ \text{m}^3/\text{sec(m)}$$

Exit gradient:

$$\Delta h = \frac{H_1 - H_2}{n_d} = \frac{27.0 - 19.5}{8} = \frac{7.5}{8}$$

$$i = \frac{\Delta h}{l} = \frac{7.5/8}{3.45} = 0.27$$



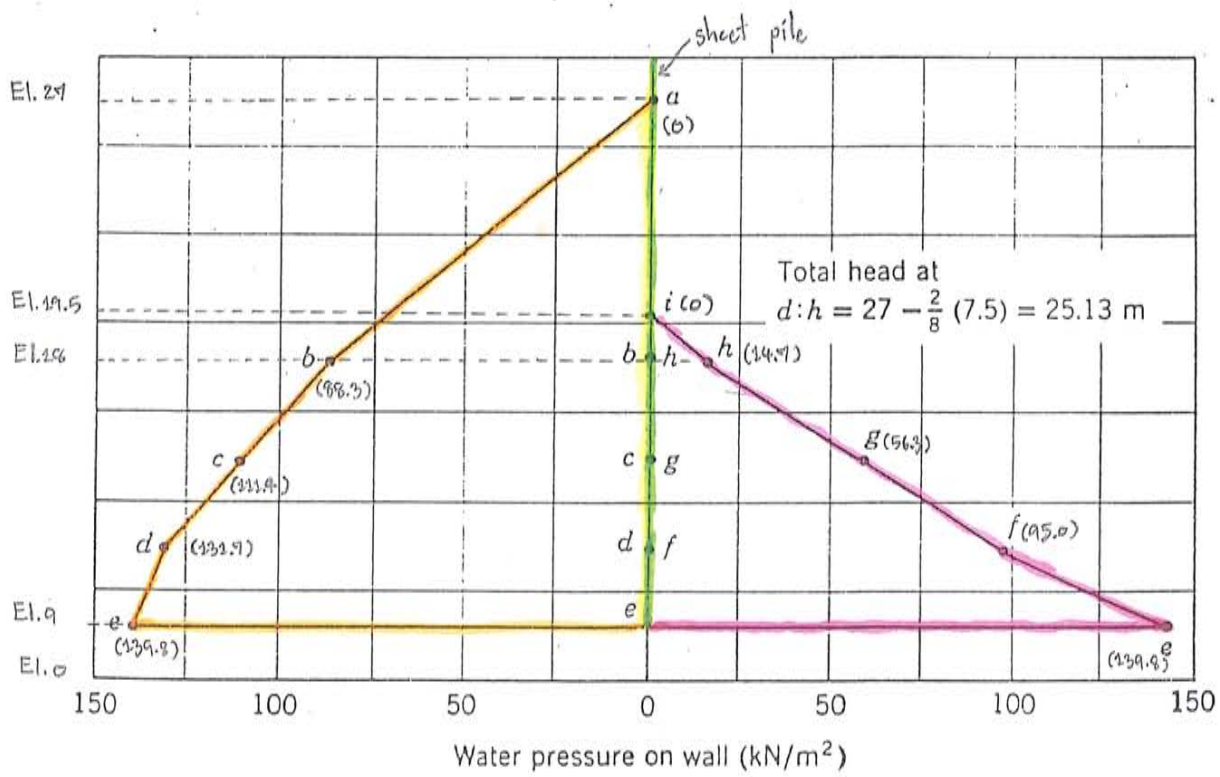


Fig. E18.1 Flow under sheet pile wall.