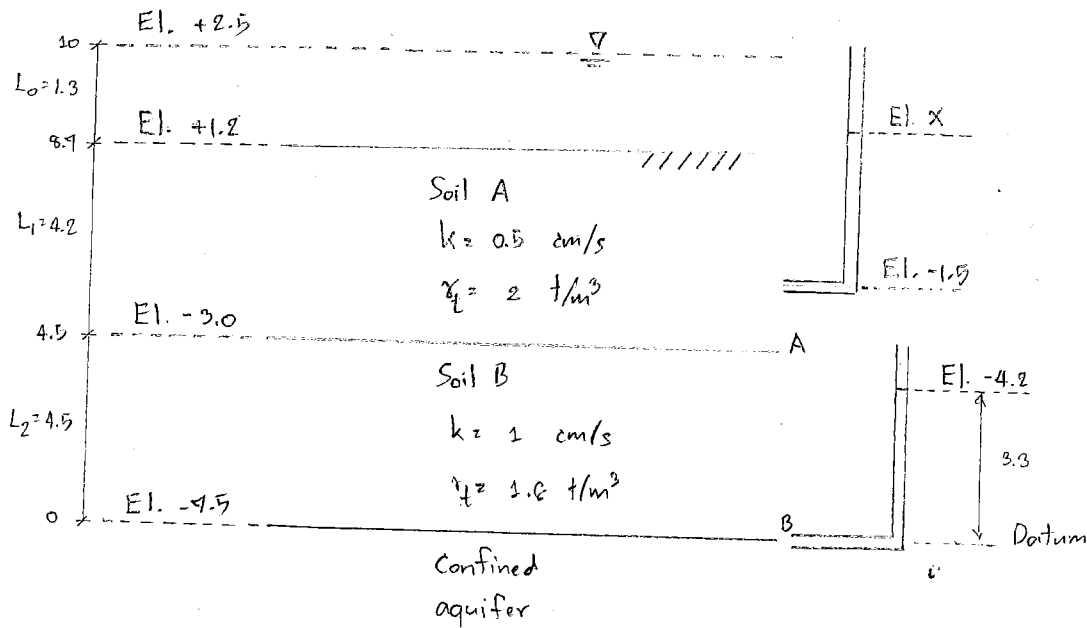


Exam 1

As shown in the figure below, the pump is installed at El. -7.5 to lower the pressure at this point.

- 1) Draw the vertical effective stress and seepage force/unit volume versus the elevation (from El. +2.5 to -7.5)
- 2) What is the rate of water that is currently pumping out?
- 3) If the pump stops, and there is an artesian occurring at the layer of confined aquifer which can cause the quick condition in the soil. What would a piezometer at El. -1.5 read Elevation "x"?



Solⁿ 1) $\Delta h = 10 - (-7.5 - 4.2) = 6.7 \text{ m.}$ (Downward flow)

and $Q_1 = Q_2$

$k_1 i_1 A_1 = k_2 i_2 A_2$; $A_1 = A_2$

$k_1 i_1 = k_2 i_2$

$k_1 \frac{\Delta h_1}{L_1} = k_2 \frac{\Delta h_2}{L_2}$

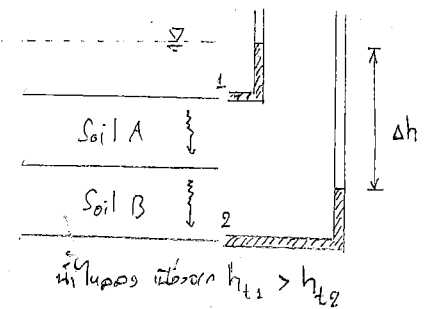
$\Delta h_1 = \frac{k_2 \times L_1}{k_1 \times L_2} \times \Delta h_2$

and $\Delta h_2 = \Delta h - \Delta h_1$

$\Delta h_1 = \frac{k_2 \times L_1}{k_1 \times L_2} \times (\Delta h - \Delta h_1)$

$\Delta h_1 = \frac{\frac{k_2}{k_1} \times \frac{L_1}{L_2} \times \Delta h}{1 + \frac{k_2}{k_1} \times \frac{L_1}{L_2}} = \frac{1 \times \frac{4.2}{4.5} \times 6.7}{1 + \frac{1}{0.5} \times \frac{4.2}{4.5}} = 4.4 \text{ m.}$

$\Delta h_2 = 6.7 - 4.4 = 2.3 \text{ m.}$



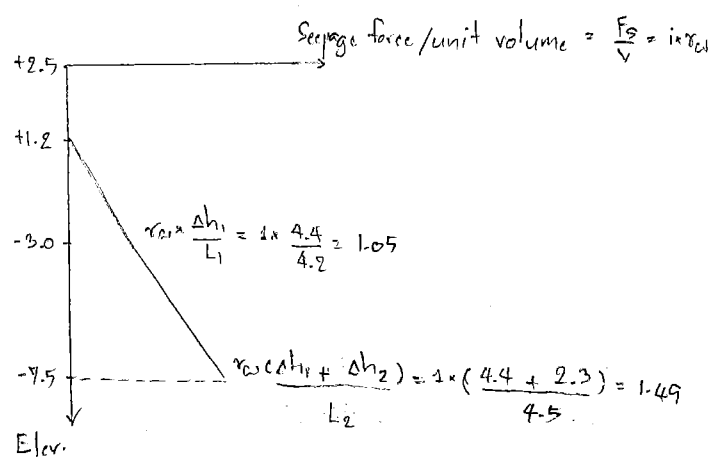
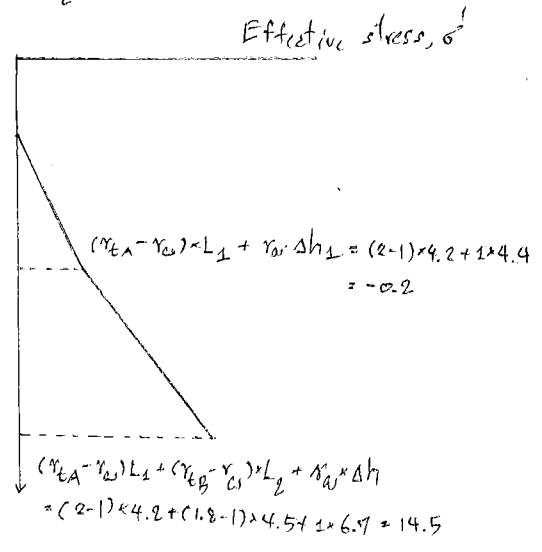
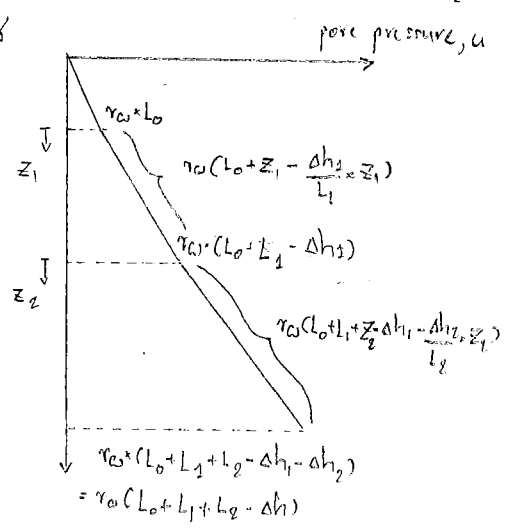
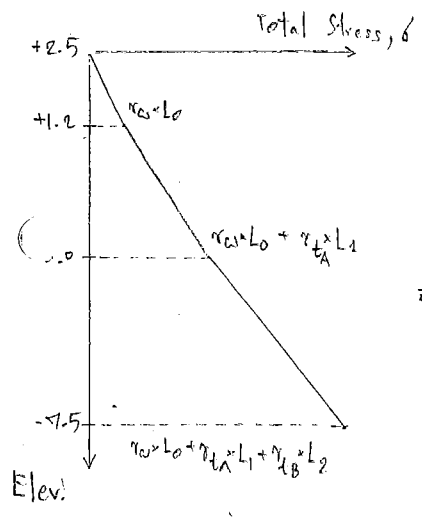
Ele.	h_c	h_e	$h_p = h_f - h_c$
+2.5	$h_c + h_p = 10 + 0 = 10$	10	0
+1.2	$h_c + h_p = 8.7 + 1.3 = 10$	8.7	1.3
-3.0	$h - \Delta h_1 = 10 - 4.4 = 5.6$		
-7.5	$h - (\Delta h_1 + \Delta h_2) = 10 - (4.4 + 2.3) = 3.3$		

$$i_a = \frac{\Delta h_A}{L_1} = \frac{(L_0 + L_1 - h_p)}{L_1} \Rightarrow h_p = (L_0 + L_1) - \Delta h_A ; \Delta h_A = \Delta h_1$$

$$= (1.3 + 4.2) - 4.4 = 1.1$$

$$i_b = \frac{\Delta h_B}{L_2} = \frac{(L_0 + L_1 + L_2 - h_p)}{L_2} \Rightarrow h_p = (L_0 + L_1 + L_2) - \Delta h_B ; \Delta h_B = \Delta h_1 + \Delta h_2$$

$$= (1.3 + 4.2 + 4.5) - 6.7 = 3.3$$



2.) $Q_{outflow} = k \cdot A \cdot i$
 $= k_B \cdot A_B \cdot i_B ; i_B = \frac{\Delta h_1 + \Delta h_2}{L_2}$
 $= (1 \times 10^{-2}) \times 1 \times 1.49$
 $= 1.49 \times 10^{-2} \text{ m}^3/\text{s}$

3) quick condition at Elev. -7.5 ($s' = 0$)

③

$$\begin{aligned}h_p \text{ at Elev. } -1.5 &= L_0 + Z_1 - \frac{\Delta h_1}{L_1} \times Z_1 \\&= 1.3 + 4.0 - \frac{4.4}{4.2} \times 4.0 \\&= 1.1 \text{ m}\end{aligned}$$

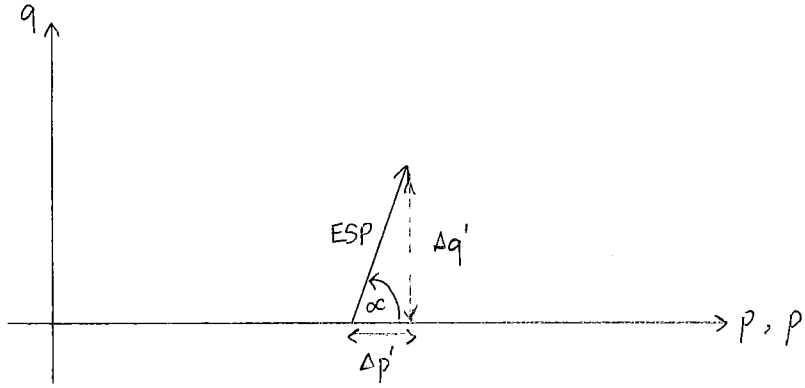
\therefore piezometer at Elev. -1.5 read Elev. $x = -1.5 + 1.1 = -0.4$

Given: 1. Triaxial Test CIUC(L) : σ_c , σ_c' , u_c and $\Delta\sigma_v$

2. The effective stress path (ESP) during shearing moves as a straight line making an angle of " α "

Find: 1. derive an expression of A-parameter as a function of these variables.

2. calculate A-parameter if $\alpha = 30^\circ, 60^\circ, 120^\circ, 150^\circ$



Consolidate \rightarrow Isotropically $\Delta\sigma_v = 0$

$$\sigma_c' = \sigma_c - u_b^{\uparrow 0}$$

$$\sigma_c' = \sigma_c$$

Undrain Compression loading $\Delta\sigma_h = 0$ during shearing ($\Delta\sigma_v > 0, \Delta u = u_c$)

visd

$$\tan \alpha = \frac{\Delta q'}{\Delta p'}$$

$$= \frac{(\Delta\sigma_v' - \Delta\sigma_h')/2}{(\Delta\sigma_v' + \Delta\sigma_h')/2} = \frac{\Delta\sigma_v' - \Delta\sigma_h'}{\Delta\sigma_v' + \Delta\sigma_h'}$$

then

$$\Delta u = B[\Delta\sigma_h + A(\Delta\sigma_v - \Delta\sigma_h)]$$

Isotropically consolidate \Rightarrow Sample is saturated $\Rightarrow B = 1.0$

$$\Delta u = \Delta\sigma_h + A(\Delta\sigma_v - \Delta\sigma_h)$$

So that

$$\Delta\sigma_v' = \Delta\sigma_v - \Delta u = \Delta\sigma_v - [\Delta\sigma_h + A(\Delta\sigma_v - \Delta\sigma_h)] = \Delta\sigma_v - \Delta\sigma_h - A(\Delta\sigma_v - \Delta\sigma_h)$$

$$\Delta\sigma_h' = \Delta\sigma_h - \Delta u = \Delta\sigma_h - [\Delta\sigma_h + A(\Delta\sigma_v - \Delta\sigma_h)] = \Delta\sigma_h - \Delta\sigma_h - A(\Delta\sigma_v - \Delta\sigma_h) = -A(\Delta\sigma_v - \Delta\sigma_h)$$

$$\Delta\sigma_v' - \Delta\sigma_h' = \Delta\sigma_v - \Delta\sigma_h - A(\Delta\sigma_v - \Delta\sigma_h) + A(\Delta\sigma_v - \Delta\sigma_h) = \Delta\sigma_v - \Delta\sigma_h$$

$$\Delta\sigma_v' + \Delta\sigma_h' = \Delta\sigma_v - \Delta\sigma_h - A(\Delta\sigma_v - \Delta\sigma_h) - A(\Delta\sigma_v - \Delta\sigma_h) = (1 - 2A)(\Delta\sigma_v - \Delta\sigma_h)$$

So

$$\tan \alpha = \frac{(\Delta\sigma_v' - \Delta\sigma_h')}{(1 - 2A)(\Delta\sigma_v' - \Delta\sigma_h')} = \frac{1}{1 - 2A}$$

$$A = \frac{1}{2} \left(1 - \frac{1}{\tan \alpha} \right) = \frac{\tan \alpha - 1}{2 \tan \alpha}$$

If $\alpha = 30^\circ, 60^\circ, 120^\circ, 150^\circ \Rightarrow A = -0.37, 0.21, 0.99, 1.37$

Exam 3

⑤

The soft clay deposit, having $\gamma_s = 1.6 \text{ t/m}^3$ and the ground water table is at the surface, has the thickness of 10 m. A point in the soil mass, located at 8 m below the ground surface and having the in-situ pore pressure of 6 t/m^2 and $K_0 = 0.8$ is subjected to the stress condition of extension loading. At failure, the value of undrained shear strength is 2.04 and $A_f = 0.6$.

Find the piezometer head at depth of 8 m.

Solⁿ 1.) before shearing (at K_0 -line)

$$\delta_{vc} = \gamma_s \times Z = 1.6 \times 8 = 12.8 \text{ t/m}^2 \quad (\text{in-situ } u = 6 \text{ t/m}^2)$$

$$\delta_{ve}' = \delta_{vc} - u = 12.8 - 6 = 6.8 \text{ t/m}^2$$

$$\delta_{hc}' = K_0 \times \delta_{ve}' = 0.8 \times 6.8 = 5.44 \text{ t/m}^2$$

$$\delta_{hc} = \delta_{hc}' + u = 5.44 + 6 = 11.44 \text{ t/m}^2$$

Total Stress Path (TSP)

$$p = \frac{\delta_{vc} + \delta_{hc}}{2} = \frac{12.8 + 11.44}{2} = 12.12$$

$$q = \frac{\delta_{vc} - \delta_{hc}}{2} = \frac{12.8 - 11.44}{2} = 0.68$$

Effective Stress Path (ESP)

$$p' = \frac{\delta_{ve}' + \delta_{hc}'}{2} = \frac{6.8 + 5.44}{2} = 6.12$$

$$q' = \frac{\delta_{ve}' - \delta_{hc}'}{2} = \frac{6.8 - 5.44}{2} = 0.68$$

2.) ΔK_0 UE (L) \Rightarrow failure, $\Delta \delta_v = \Delta \delta_3 = 0$, $\Delta \delta_h = \Delta \delta_1 > 0$

$$c_u = 2.04 \text{ t/m}^2$$

At failure $q_f = c_u = 2.04 \text{ t/m}^2$

$$\Delta \delta_v = \Delta \delta_3 = \delta_{vf} - \delta_{vc}, \quad \Delta \delta_h = \Delta \delta_1 = \delta_{hf} - \delta_{hc}$$

$$0 = \delta_{vf} - \delta_{vc}$$

$$\delta_{vf} = \delta_{vc} = 11.44$$

$$\Delta \delta_h = 15.52 - 11.44$$

$$= 4.08 \text{ t/m}^2$$

$$q_f = c_u = \frac{\delta_{vf} - \delta_{hf}}{2}$$

$$-2.04 = \frac{11.44 - \delta_{hf}}{2}$$

$$\delta_{hf} = 15.52 \text{ t/m}^2$$

$$\Delta u = B [\Delta \delta_3 + A_f (\Delta \delta_1 - \Delta \delta_3)]$$

$$= 1.0 [0 + 0.6 (4.08 - 0)]$$

$$= 0.648 \text{ t/m}^2$$

pore pressure at failure, $u_f = u + \Delta u = 6 + 0.648 = 6.648 \text{ t/m}^2$, $h_p = \frac{u_f}{\gamma_w} = 6.648 \text{ m}$.

$$\delta_{vf}' = \delta_{vf} - u_f = 11.44 - 6.648 = 4.792 \text{ t/m}^2$$

$$\delta_{hf}' = \delta_{hf} - u_f = 15.52 - 6.648 = 8.872 \text{ t/m}^2$$

Total Stress Path (TSP)

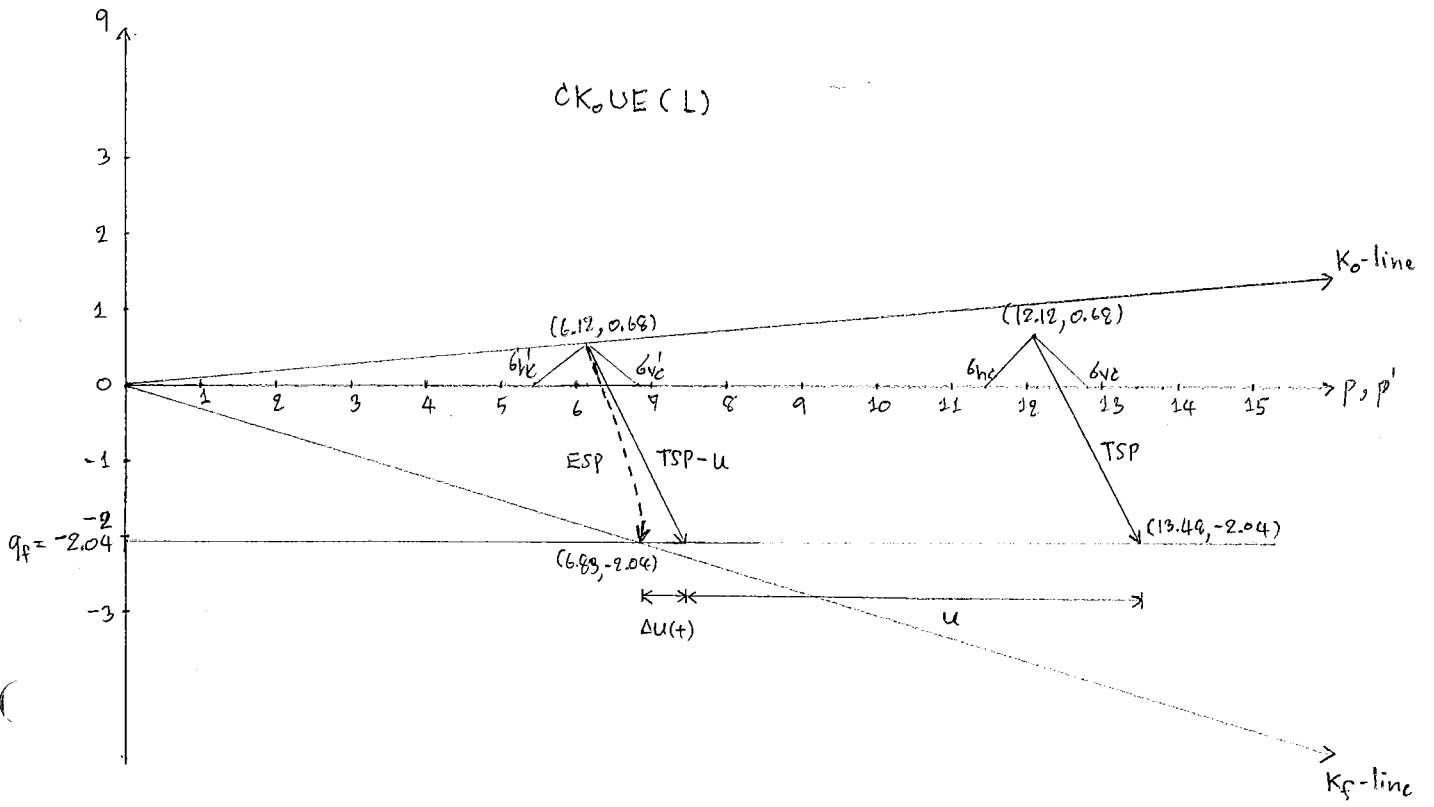
$$p_f = \frac{\delta_{vf}' + \delta_{hf}'}{2} = \frac{4.792 + 8.872}{2} = 13.48$$

$$q_f = \frac{\delta_{vf}' - \delta_{hf}'}{2} = \frac{4.792 - 8.872}{2} = -2.04$$

Effective Stress Path (ESP)

$$p_f' = \frac{\delta_{vf}' + \delta_{hf}'}{2} = \frac{4.792 + 8.872}{2} = 6.83$$

$$q_f' = \frac{\delta_{vf}' - \delta_{hf}'}{2} = \frac{4.792 - 8.872}{2} = -2.04$$

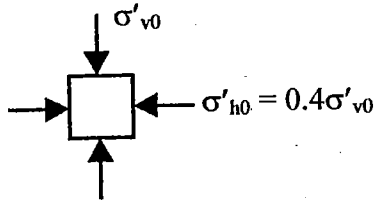


p - q Diagram

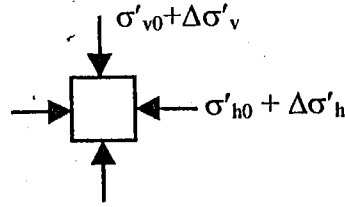
Question 1: (10 points)

A soil obeys the Mohr-Coulomb failure criterion, in which the effective stress parameters are: $a' = 0.7 \text{ t/m}^2$, $\alpha' = 28^\circ$. The soil begins with an anisotropic state of stress as shown in the figure below. The increment effective stresses are applied such that: $\Delta\sigma'_h = -\Delta\sigma'_v/4$, where $\Delta\sigma'_v > 0$, until the soil fails.

- 1) Draw the stress path in p-q diagram and mark the point of failure on K_f line
- 2) Calculate the shear and normal stresses on the failure plane and the orientation of the failure plane.



Initial stress



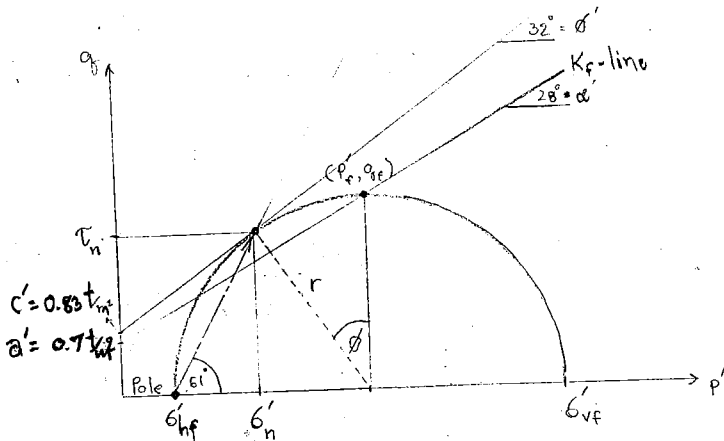
Soil subject to applied stresses

$$P'_0 = \frac{\sigma'_{v0} + \sigma'_{h0}}{2} = \frac{\sigma'_{v0} + 0.4\sigma'_{v0}}{2} = 0.7\sigma'_{v0}$$

$$\Delta P' = \frac{\Delta\sigma'_v + \Delta\sigma'_h}{2} = \frac{\Delta\sigma'_v + (-\frac{\Delta\sigma'_v}{4})}{2} = \frac{3\Delta\sigma'_v}{8}$$

$$q_0 = \frac{\sigma'_{v0} - \sigma'_{h0}}{2} = \frac{\sigma'_{v0} - 0.4\sigma'_{v0}}{2} = 0.3\sigma'_{v0}$$

$$\Delta q = \frac{\Delta\sigma'_v - \Delta\sigma'_h}{2} = \frac{\Delta\sigma'_v - (-\frac{\Delta\sigma'_v}{4})}{2} = \frac{5\Delta\sigma'_v}{8}$$



จุดบน Envelope คือ จุดที่ความเค้นเฉือนคือ

$$a' + p' \tan \alpha' = q$$

$$0.7 + (P'_0 + \Delta P') \tan 28^\circ = q_0 + \Delta q \quad (*)$$

แทน $P'_0, q_0, \Delta P', \Delta q$ ใน (*) จะได้สมการดังนี้

$$(0.7 \tan 28^\circ - 0.3) \sigma'_{v0} + \left(\frac{3 \tan 28^\circ}{8} - \frac{5}{8} \right) \Delta \sigma'_v + 0.7 = 0$$

$$\therefore \Delta \sigma'_v = - \left[\frac{0.7 + (0.7 \tan 28^\circ - 0.3) \sigma'_{v0}}{\left(\frac{3 \tan 28^\circ}{8} - \frac{5}{8} \right)} \right]$$

$$= - \left(\frac{0.7 + 0.072 \sigma'_{v0}}{(-0.426)} \right) = 1.64 + 0.17 \sigma'_{v0}$$

ดังนั้น $\Delta P' = \frac{3\Delta\sigma'_v}{8} = 0.615 + 0.064 \sigma'_{v0}$

ดังนั้น $\Delta q = \frac{5\Delta\sigma'_v}{8} = 1.025 + 0.106 \sigma'_{v0}$

ที่ Failure $P'_f = P'_0 + \Delta P' = 0.7\sigma'_{v0} + (0.615 + 0.064 \sigma'_{v0}) = 0.615 + 0.764 \sigma'_{v0}$

$q_f = q_0 + \Delta q = 0.3\sigma'_{v0} + (1.025 + 0.106 \sigma'_{v0}) = 1.025 + 0.406 \sigma'_{v0}$

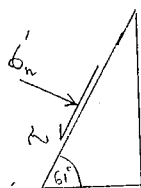
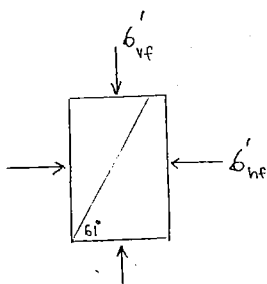
$$\phi' = \sin^{-1}(\tan \alpha') = \sin^{-1}(\tan 28^\circ) = 32^\circ$$

$$c' = \frac{a'}{\cos \phi'} = \frac{0.7}{\cos 32^\circ} = 0.83 \text{ t/m}^2$$

$$r = q_f$$

$$\sigma'_n = P'_f - r \sin \phi' = 0.072 - 0.549 \sigma'_{v0}$$

$$\tau = r \cos \phi' = 0.869 + 0.344 \sigma'_{v0}$$



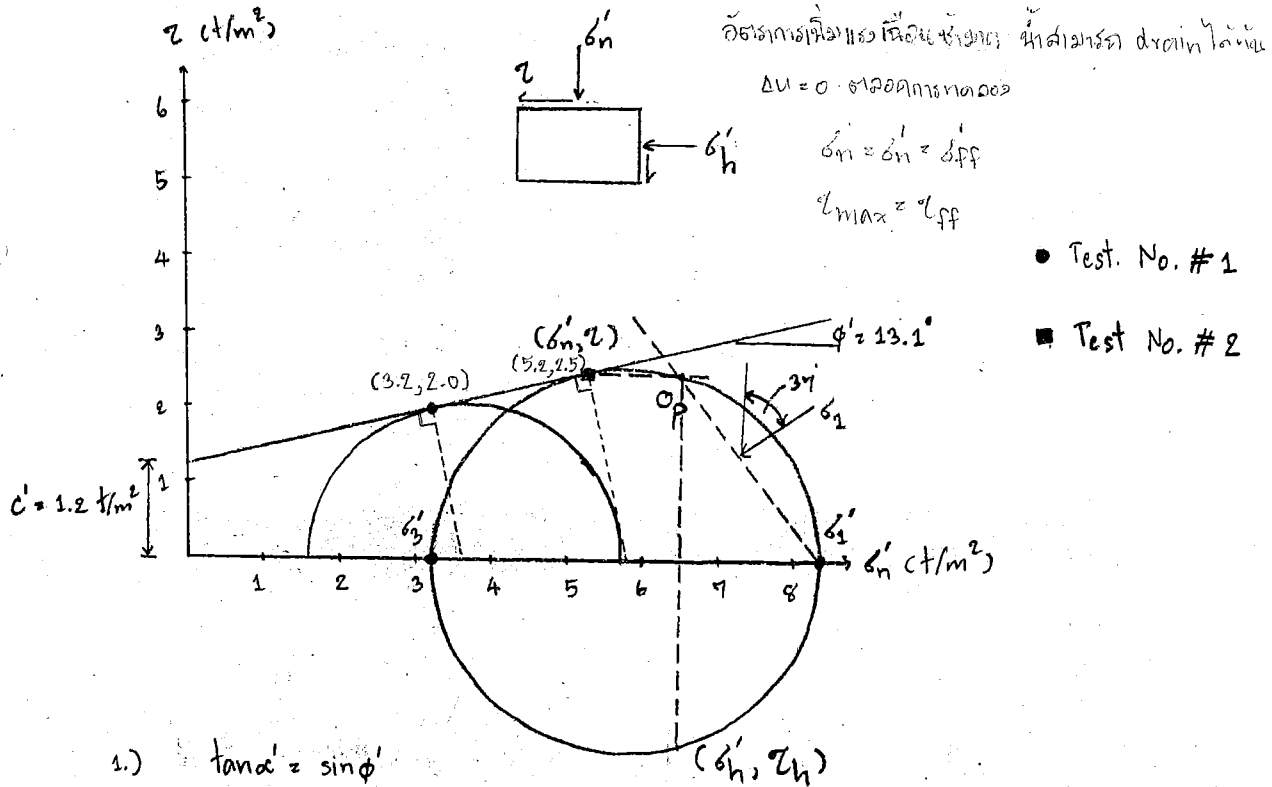
Question 2: (10 points)

The followings are the results at failure of the direct shear test on a dry sand:

Test No.	Normal stress t/m ²	Shear stress t/m ²
#1	3.2	2.0
#2	5.2	2.5

Assuming the soil obey Mohr-Coulomb failure criterion,

- 1) Calculate the effective strength parameter (c', ϕ') and (a', α')
- 2) For Test #2, calculate the magnitude of σ_1 , and σ_3 ; and the direction of σ_1 to the vertical line.
- 3) For Test #2, calculate the magnitude of horizontal stress at failure.

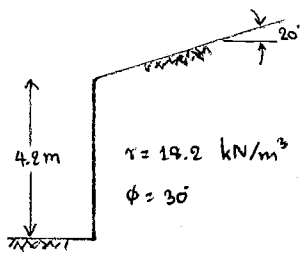


1.) $\tan \alpha' = \sin \phi'$
 $\rightarrow \alpha' = \tan^{-1}(\sin \phi') = \tan^{-1}(\sin 13.1) = 12.8^\circ$
 $\rightarrow a' = c' \cos \phi' = 1.2 \cos 13.1 = 1.2 \text{ t/m}^2$
 $c' = 1.2 \text{ t/m}^2$
 $\phi' = 13.1^\circ$

2.) จานวน $\sigma'_1 = 8.3 \text{ t/m}^2$, $\sigma'_3 = 3.3 \text{ t/m}^2$ for Test #2

หาแกนแนวราบ หรือ pole, O_p จานวน plane of σ'_1
 จานวน direction of σ'_1 to the vertical line = 34°

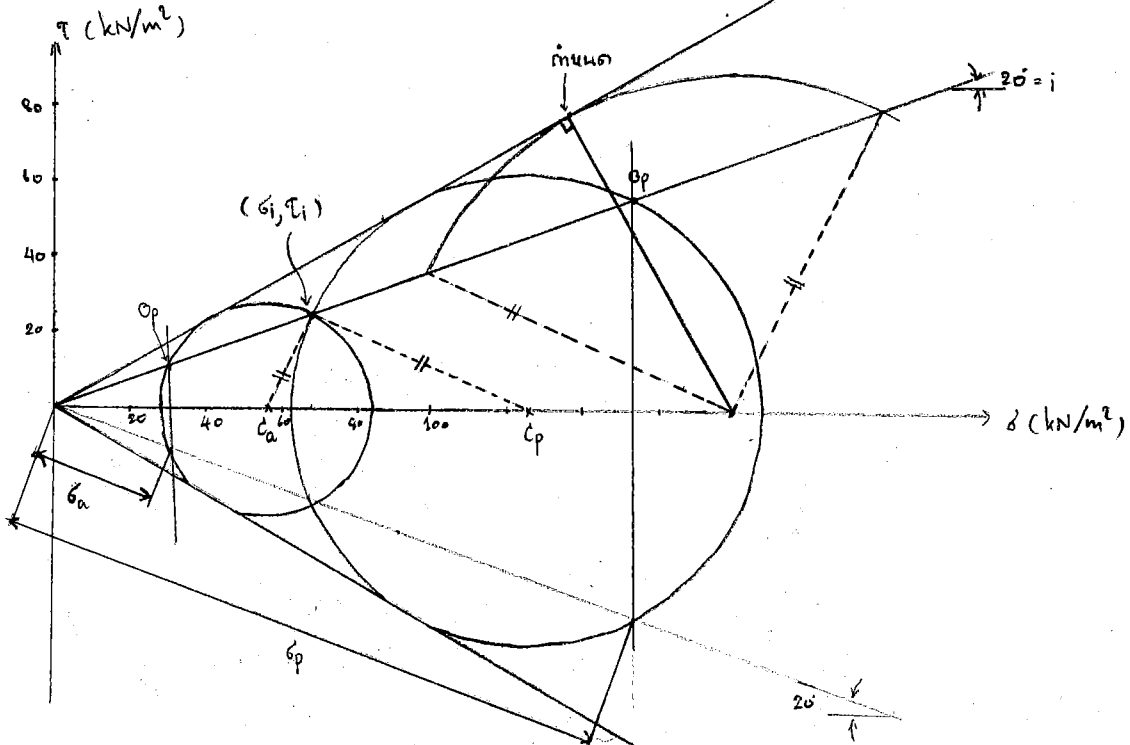
3.) หาแกนแนวตั้ง จานวน pole จานวนทิศทางของ mohr circle จานวน direction of horizontal plane
 จานวน $\sigma'_h = 6.5 \text{ t/m}^2$ for Test #2



$$\sigma_1 = \gamma \cdot H \cos i \cdot \cos i = 18.2 \cdot 4.2 \cos^2(20^\circ) = 61.50$$

$$\tau_1 = \gamma \cdot H \cos i \cdot \sin i = 18.2 \cdot 4.2 \cos(20^\circ) \cdot \sin(20^\circ) = 24.60$$

①



$$\sigma_a = 33 \text{ kN/m}^2 \Rightarrow P_a = \frac{1}{2} (33) \cdot 4.2 = 69.3 \text{ kN/m}$$

$$\sigma_p = 164 \text{ kN/m}^2 \Rightarrow P_p = \frac{1}{2} (164) \cdot 4.2 = 344.4 \text{ kN/m}$$

②

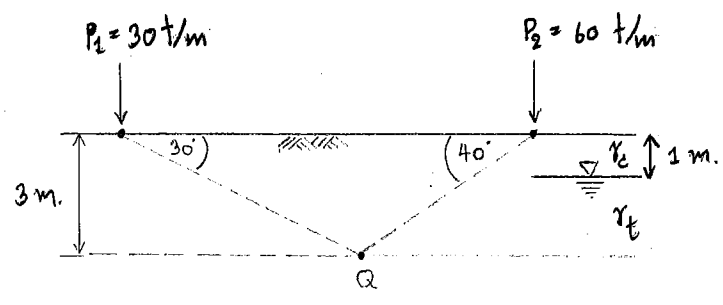
$$K_a = \frac{\cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}}{\cos 20^\circ} = \frac{\cos 20^\circ \frac{\cos 20^\circ - \sqrt{\cos^2 20^\circ - \cos^2 30^\circ}}{\cos 20^\circ + \sqrt{\cos^2 20^\circ - \cos^2 30^\circ}}}{\cos 20^\circ + \sqrt{\cos^2 20^\circ - \cos^2 30^\circ}} = \frac{0.94 \cdot 0.595}{1.305} = 0.414$$

$$\Rightarrow P_a = \frac{1}{2} \cdot K_a \cdot \gamma H^2 = \frac{1}{2} \cdot 0.414 \cdot 18.2 \cdot 4.2^2 = 66.4 \text{ kN/m}$$

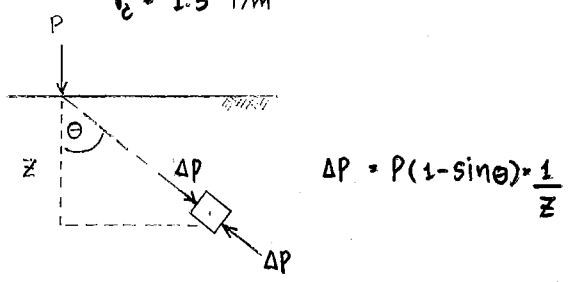
$$K_p = \frac{\cos \beta \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}}{\cos 20^\circ} = \frac{\cos 20^\circ \frac{\cos 20^\circ + \sqrt{\cos^2 20^\circ - \cos^2 30^\circ}}{\cos 20^\circ - \sqrt{\cos^2 20^\circ - \cos^2 30^\circ}}}{\cos 20^\circ - \sqrt{\cos^2 20^\circ - \cos^2 30^\circ}} = \frac{0.94 \cdot 1.305}{0.595} = 2.133$$

$$\Rightarrow P_p = \frac{1}{2} \cdot K_p \cdot \gamma H^2 = \frac{1}{2} \cdot 2.133 \cdot 18.2 \cdot 4.2^2 = 342.4 \text{ kN/m}$$

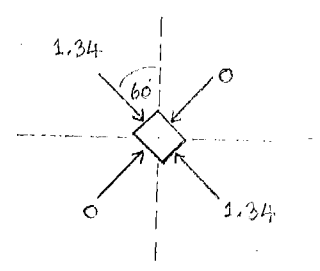
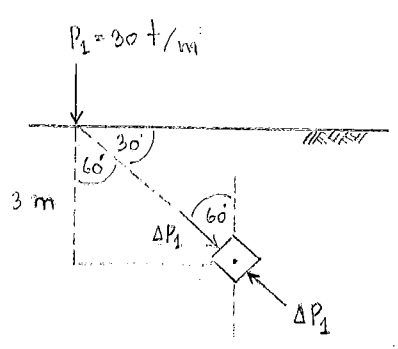
Quiz 1



Given $K_0 = 2$; อัตราการเปลี่ยน load ชั่วคราว
 $\gamma_t = 1.7 \text{ t/m}^3$
 $\gamma_c = 1.5 \text{ t/m}^3$

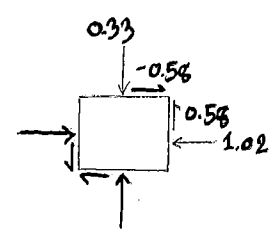
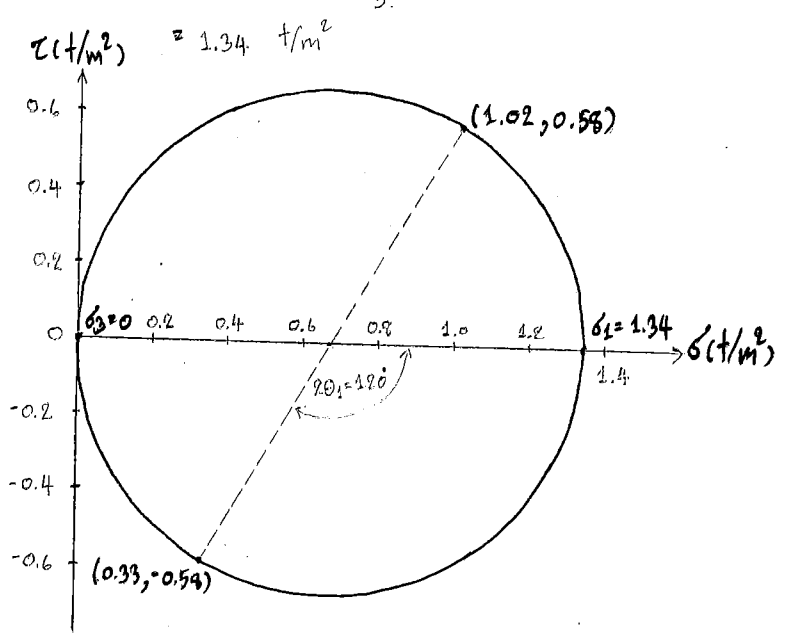


Sol¹¹ 1.) คำนวณหา $P_1 = 30 \text{ t/m}$

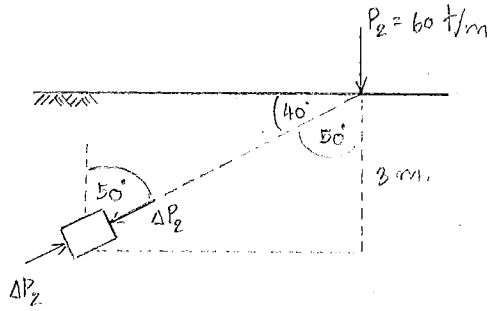


$\Delta P_1 = P_1(1 - \sin\theta_1) \cdot \frac{1}{z}$; $\theta_1 = 60^\circ$
 $= 30 \cdot (1 - \sin 60^\circ) \cdot \frac{1}{3}$

$\sigma_1 = \Delta P_1 = 1.34 \text{ t/m}^2$
 $\sigma_3 = 0$



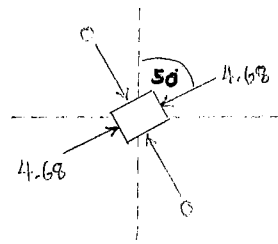
2.) $P_2 = 60 \text{ t/m}$



$$\Delta P_2 = P_2 (1 - \sin \theta_2) \times \frac{1}{Z} \quad ; \quad \theta_2 = 50^\circ$$

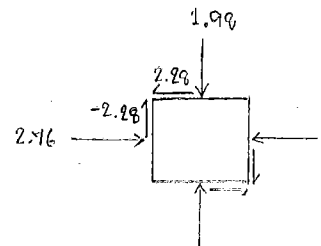
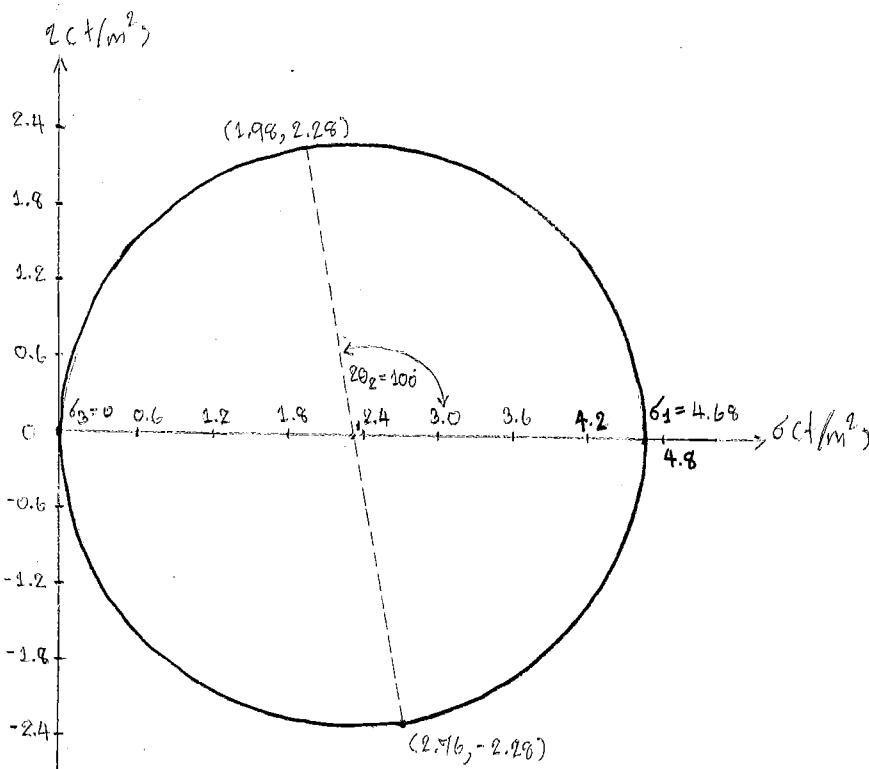
$$= 60 \times (1 - \sin 50^\circ) \times \frac{1}{3}$$

$$= 4.68 \text{ t/m}^2$$

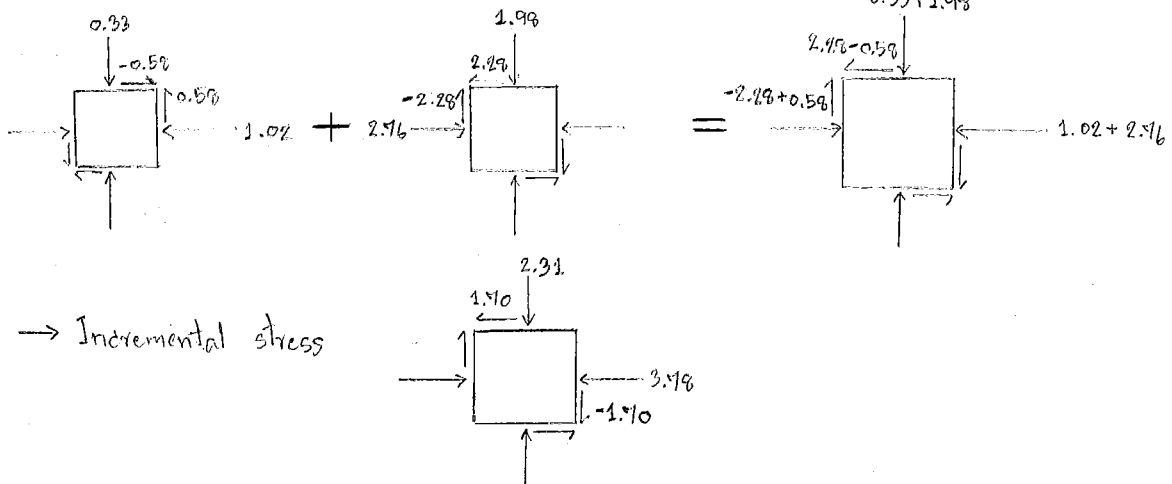


$$\sigma_1 = \Delta P_2 = 4.68 \text{ t/m}^2$$

$$\sigma_2 = 0$$



2) stress σ_1 and σ_2 in the element



3.) σ_{vo} initial state of stress in soil Q

assume pore pressure with hydrostatic

$$u_0 = \gamma_w \cdot z = 1.0 \cdot 2 = 2.0 \text{ t/m}^2$$

$$\sigma_{vo} = (1.5 \cdot 1) + (1.7 \cdot 2) = 4.9 \text{ t/m}^2$$

$$\sigma'_{vo} = \sigma_{vo} - u_0 = 4.9 - 2.0 = 2.9 \text{ t/m}^2$$

$$\sigma'_{ho} = K_0 \cdot \sigma'_{vo} = 2 \cdot 2.9 = 5.8 \text{ t/m}^2$$

$$\sigma_{ho} = \sigma'_{ho} + u_0 = 5.8 + 2.0 = 7.8 \text{ t/m}^2$$

$$p = \frac{\sigma_{vo} + \sigma_{ho}}{2} = \frac{4.9 + 7.8}{2} = 6.35$$

$$q = \frac{\sigma_{vo} - \sigma_{ho}}{2} = \frac{4.9 - 7.8}{2} = -1.45$$

$$p' = \frac{\sigma'_{vo} + \sigma'_{ho}}{2} = \frac{2.9 + 5.8}{2} = 4.35$$

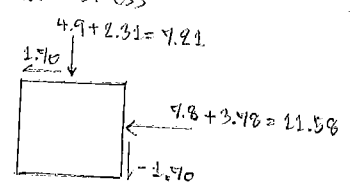
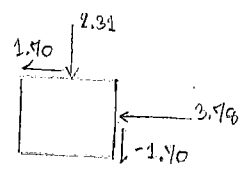
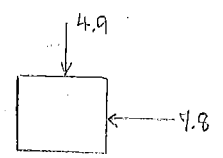
$$q' = q = \frac{\sigma'_{vo} - \sigma'_{ho}}{2} = \frac{2.9 - 5.8}{2} = -1.45$$

initial stress

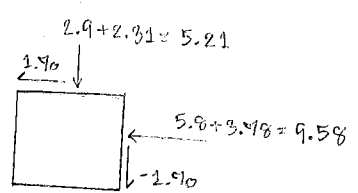
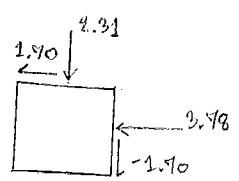
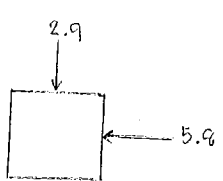
stress increment

final stress

Total stress



Effective stress



► $\Delta u = 0$

$$\Delta \sigma_v = 2.31 \rightarrow \Delta \sigma'_v = \Delta \sigma_v + \Delta u = 2.31$$

$$\Delta \sigma_h = 3.78 \rightarrow \Delta \sigma'_h = \Delta \sigma_h + \Delta u = 3.78$$

$$\Delta \tau = 1.70 \rightarrow \Delta \tau' = \Delta \tau + \Delta u = 1.70$$

► final stress

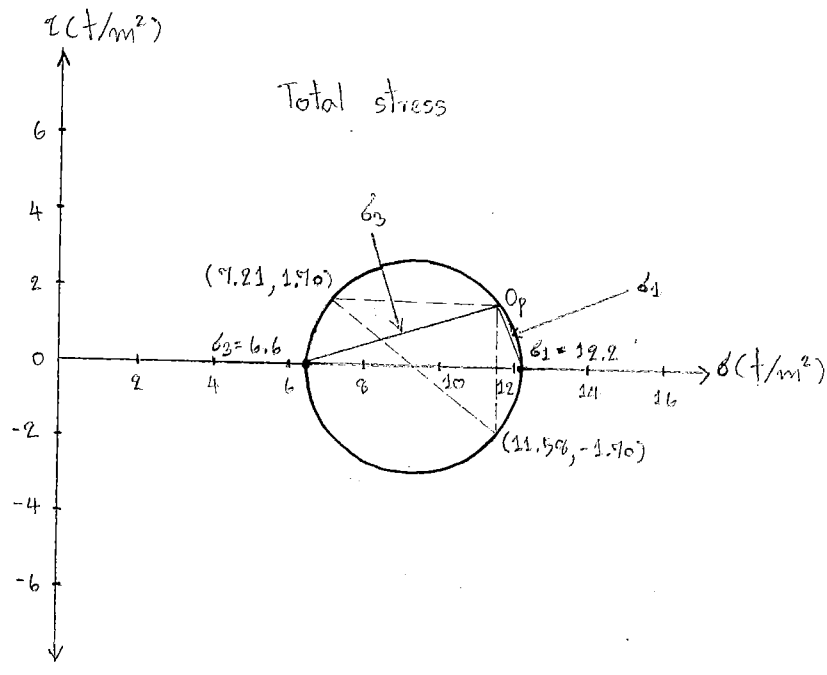
$$\sigma_{vf} = \sigma_{vo} + \Delta \sigma_v = 4.9 + 2.31 = 7.21 \text{ t/m}^2$$

$$\sigma_{hf} = \sigma_{ho} + \Delta \sigma_h = 7.8 + 3.78 = 11.58 \text{ t/m}^2$$

$$\sigma'_{vf} = \sigma'_{vo} + \Delta \sigma'_v = 2.9 + 2.31 = 5.21 \text{ t/m}^2$$

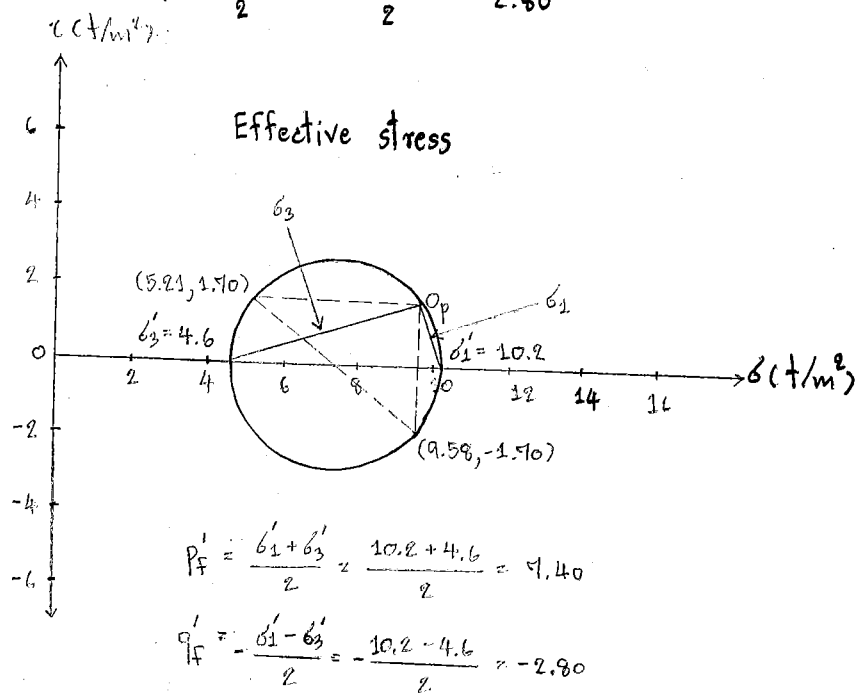
$$\sigma'_{hf} = \sigma'_{ho} + \Delta \sigma'_h = 5.8 + 3.78 = 9.58 \text{ t/m}^2$$

$$\tau_f = \Delta \tau = 1.70 \text{ t/m}^2$$



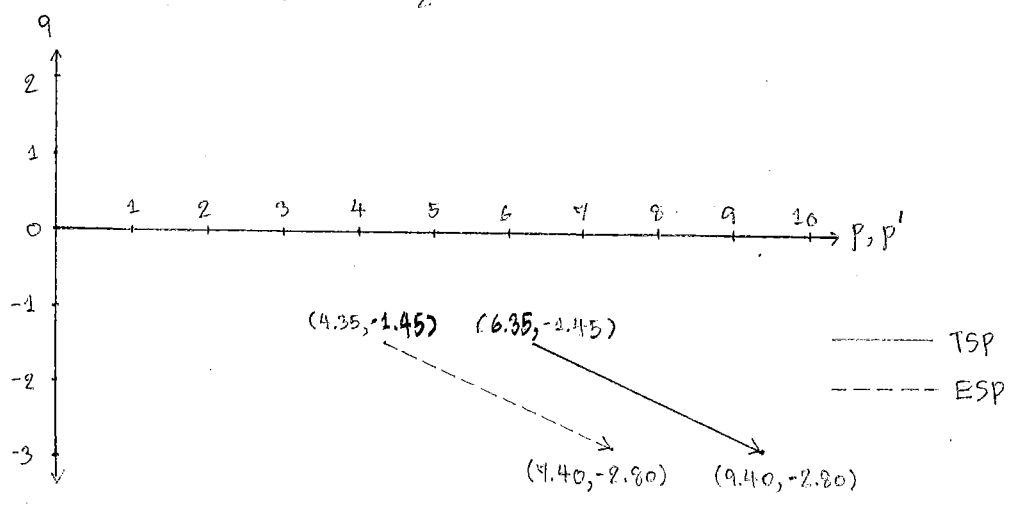
$$p_F = \frac{\sigma_1 + \sigma_3}{2} = \frac{12.2 + 6.6}{2} = 9.40$$

$$q_F = \frac{\sigma_1 - \sigma_3}{2} = \frac{12.2 - 6.6}{2} = 2.80$$



$$p_F' = \frac{\sigma_1' + \sigma_3'}{2} = \frac{10.2 + 4.6}{2} = 7.40$$

$$q_F' = \frac{\sigma_1' - \sigma_3'}{2} = \frac{10.2 - 4.6}{2} = 2.80$$



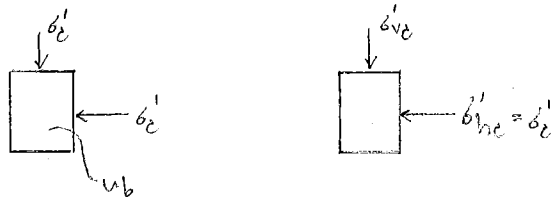
Quiz 2

The anisotropically consolidated triaxial extension test on normally consolidated clay yields the value of $\frac{c_u}{\delta'_{vc}} = 0.22$ and $A_f = 0.9$. The value of $\delta_c = 30 \text{ t/m}^2$,

During consolidation the value of $K = 0.5$. Estimated the followings:

- (a) The excess pore pressure at failure when the total stress path during shear is unloading extension. The back pressure is 20 t/m^2
- (b) The value of ϕ' for normally consolidated clay
- (c) The ratio of $\left(\frac{\delta'_1}{\delta'_3}\right)$ at q_f condition

Solⁿ 1.) At the end of consolidation : before shearing



$$\delta'_c = \delta_c - u_b = 30 - 20 = 10 \text{ t/m}^2 \quad ; \quad \delta'_{hc} = \delta'_c = 10 \text{ t/m}^2$$

$$K_0 = \frac{\delta'_{hc}}{\delta'_{vc}} \implies \delta'_{vc} = \frac{\delta'_{hc}}{K_0} = \frac{10}{0.5} = 20 \text{ t/m}^2$$

$$\delta'_{hc} = \delta'_{hc} + u = \delta'_{hc} + (u_b + \Delta u) = 10 + (20 + 0) = 30 \text{ t/m}^2$$

$\Delta u = 0$ Reason: end of consolidation before shearing

$$\delta'_{vc} = \delta'_{vc} + u = \delta'_{vc} + (u_b + \Delta u) = 20 + (20 + 0) = 40 \text{ t/m}^2$$

► initial shear stress, q

$$p = \frac{\delta'_{vc} + \delta'_{hc}}{2} = \frac{40 + 30}{2} = 35$$

$$p' = \frac{\delta'_{vc} + \delta'_{hc}}{2} = \frac{20 + 10}{2} = 15$$

$$q = \frac{\delta'_{vc} - \delta'_{hc}}{2} = \frac{40 - 30}{2} = 5$$

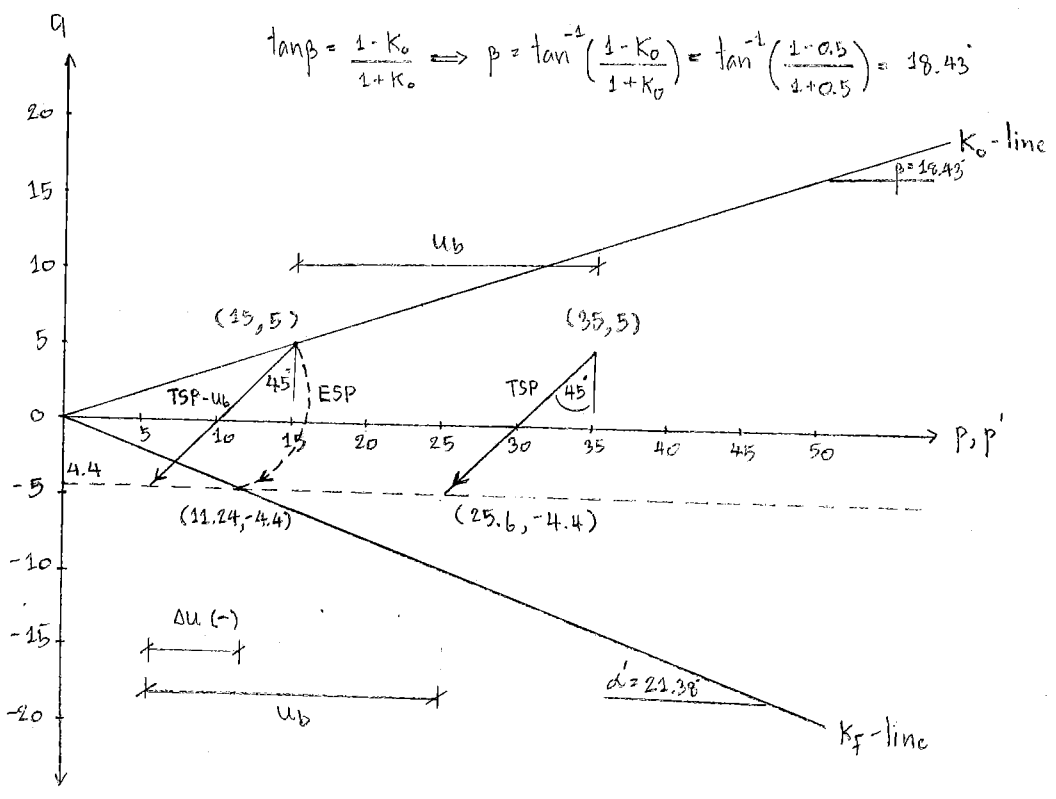
$$q' = q = \frac{\delta'_{vc} - \delta'_{hc}}{2} = \frac{20 - 10}{2} = 5$$

2.) max shearing ratio failure ϕ' in triaxial extension unloading test

$$\frac{c_u}{\delta'_{vc}} = 0.22 \implies c_u = 0.22 \times \delta'_{vc} = 0.22 \times 20 = 4.4 \text{ t/m}^2$$

at failure $q_f = c_u = 4.4 \text{ t/m}^2$

draw p - q diagram



$$\tan \beta = \frac{1 - K_0}{1 + K_0} \Rightarrow \beta = \tan^{-1} \left(\frac{1 - K_0}{1 + K_0} \right) = \tan^{-1} \left(\frac{1 - 0.5}{1 + 0.5} \right) = 18.43^\circ$$

TSP $q_f(p_f, q_f) = (25.6, -4.4)$

▶ since triaxial extension unloading test

$$\Delta \sigma_1 = \Delta \sigma_2 = 0, \quad \Delta \sigma_3 = \Delta \sigma_3 < 0$$

$$\Delta u = B [\Delta \sigma_3 + A (\Delta \sigma_1 - \Delta \sigma_3)] \quad ; \quad B = 4 \text{ for saturated clay in triaxial test}$$

in failure

$$\Delta u = \Delta \sigma_3 + A (-\Delta \sigma_3) \quad ; \quad \Delta \sigma_3 = \sigma_{vf} - \sigma_{vc}, \quad \Delta \sigma_1 = \sigma_{hf} - \sigma_{hc} = 0 \Rightarrow \sigma_{hf} = \sigma_{hc} = 30 \text{ t/m}^2$$

$$\Delta u = -18.9 + 0.7 \cdot (18.9) \quad p_f = \frac{\sigma_{vf} + \sigma_{hf}}{2} = 25.6 \Rightarrow \frac{\sigma_{vf} + \sigma_{hc}}{2} = 25.6$$

$$= -5.64 \text{ t/m}^2$$

$$\frac{\sigma_{vf} + 30}{2} = 25.6 \Rightarrow \sigma_{vf} = 21.2 \text{ t/m}^2$$

$$\Delta \sigma_3 = 21.2 - 40 = -18.8 \text{ t/m}^2$$

$$\text{ESP } q_f(p_f', q_f') = (p_f - u_b - \Delta u, -4.4) = (25.6 - 20 - (-5.64), -4.4) = (11.24, -4.4)$$

▶ on K_f -line since q_f ESP $q_f(p_f', q_f')$ active

$$\alpha' = \tan^{-1} \left(\frac{q_f'}{p_f'} \right) = \tan^{-1} \left(\frac{4.4}{11.24} \right) = 21.38^\circ$$

$$\sin \phi' = \tan \alpha' \Rightarrow \phi' = \sin^{-1} (\tan \alpha') = \sin^{-1} (\tan 21.38^\circ) = 23.05^\circ$$

$$p_f' = \frac{\sigma_{1f}' + \sigma_{3f}'}{2} = 11.24 \quad (1)$$

$$q_f' = \frac{\sigma_{1f}' - \sigma_{3f}'}{2} = 4.4 \quad (2)$$

$$(1) + (2) \Rightarrow \sigma_{1f}' = 11.24 + 4.4 = 15.64 \text{ t/m}^2$$

$$(1) - (2) \Rightarrow \sigma_{3f}' = 11.24 - 4.4 = 6.84 \text{ t/m}^2$$

$$\left(\frac{\sigma_{1f}'}{\sigma_{3f}'} \right) = \frac{15.64}{6.84} = 2.29$$

$$\Delta u = \sigma_{3f}' - \sigma_{1f}' - u_b = 6.84 - 15.64 - 20 = -18.8 \text{ t/m}^2 \quad \text{OK}$$

Quiz 3

In the K_0 consolidated undrained triaxial extension test (loading), using effective consolidation stress before shearing $\sigma_c = 30 \text{ t/m}^2$ and the back pressure of 20 t/m^2 yield the results as follows :

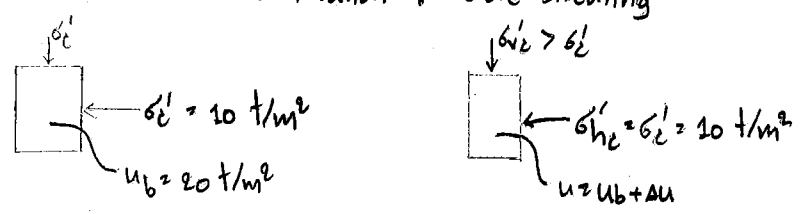
- (i) $\frac{c_u}{\sigma'_{vc}} = 0.8$, $K_0 = 0.6$
- (ii) $A_f = 0.9$
- (iii) $\phi' = 28^\circ$, $c' = 0$

Find the values of

- (a) initial shear stress
- (b) the value of pore pressure at failure
- (c) the ratio of $\frac{\sigma'_{1f}}{\sigma'_{3f}}$

Solⁿ

1.) At the end of consolidation : before shearing



$$\sigma_c = \sigma'_c + u_b \implies \sigma'_c = \sigma_c - u_b = 30 - 20 = 10 \text{ t/m}^2, \quad \sigma'_{hc} = \sigma'_e = 10 \text{ t/m}^2$$

$$K_0 = \frac{\sigma'_{hc}}{\sigma'_{vc}} \implies \sigma'_{vc} = \frac{\sigma'_{hc}}{K_0} = \frac{10}{0.6} = 16.67 \text{ t/m}^2$$

$$\sigma_{hc} = \sigma'_{hc} + u = \sigma'_{hc} + (u_b + \Delta u) = 10 + (20 + 0) = 30 \text{ t/m}^2$$

$\Delta u = 0$ at the end of consolidation before shearing

► initial shear stress, q

$$\sigma_{vc} = \sigma'_{vc} + u = \sigma'_{vc} + (u_b + \Delta u) = 16.67 + (20 + 0) = 36.67 \text{ t/m}^2$$

$$p = \frac{\sigma_{vc} + \sigma_{hc}}{2} = \frac{36.67 + 30}{2} = 33.34$$

$$p' = \frac{\sigma'_{vc} + \sigma'_{hc}}{2} = \frac{16.67 + 10}{2} = 13.34$$

$$q = \frac{\sigma_{vc} - \sigma_{hc}}{2} = \frac{36.67 - 30}{2} = 3.34$$

$$q' = q = \frac{\sigma'_{vc} - \sigma'_{hc}}{2} = \frac{16.67 - 10}{2} = 3.34$$

2.) same shearing ratio failure σ_{vc} in triaxial extension loading test

$$\frac{c_u}{\sigma'_{vc}} = 0.8 \implies c_u = 0.8 \times \sigma'_{vc} = 0.8 \times 16.67 = 13.34 \text{ t/m}^2$$

at failure $q_f = c_u = 13.34 \text{ t/m}^2$

$$\tan \alpha' = \sin \phi' \implies \alpha' = \tan^{-1}(\sin \phi') = \tan^{-1}(\sin 28^\circ) = 25.15^\circ$$

$$\tan \alpha' = \tan 25.15^\circ = 0.47$$

$$\frac{q_f}{p'_f} = \tan \alpha' \implies p'_f = \frac{q_f}{\tan \alpha'} = \frac{13.34}{0.47} = 28.38 \text{ t/m}^2$$

$$p'_f = \frac{\delta'_{1f} + \delta'_{3f}}{2} = 28.38 \quad (1)$$

$$q_f = \frac{\delta'_{1f} - \delta'_{3f}}{2} = 13.34 \quad (2)$$

$$(1) + (2) \implies \delta'_{1f} = 28.38 + 13.34 = 41.72 \text{ t/m}^2$$

$$(1) - (2) \implies \delta'_{3f} = 28.38 - 13.34 = 15.04 \text{ t/m}^2$$

$$\frac{\delta'_{1f}}{\delta'_{3f}} = \frac{41.72}{15.04} = 2.77$$

► since triaxial extension (loading) test

$$\delta'_{1f} = \delta'_{hf} = 41.72 \text{ t/m}^2$$

$$p'_f = 28.38$$

$$\delta'_{3f} = \delta'_{vf} = 15.04 \text{ t/m}^2$$

$$q_f = -13.34$$

► at failure, $\Delta u = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]$

$$\Delta \sigma_3 = \delta'_{vf} - \delta'_{vc} \implies \delta'_{vf} = \delta'_{vc} + 20 \text{ t/m}^2$$

triaxial extension loading $\Delta \sigma'_v = \Delta \sigma'_3 = 0$, $\Delta \sigma'_h = \Delta \sigma'_1 > 0$

$$\Delta \sigma'_1 = \delta'_{hf} - \delta'_{hc} \quad \left| \quad p'_f = \frac{\delta'_{vf} + \delta'_{hf}}{2} = 50.00 \right.$$

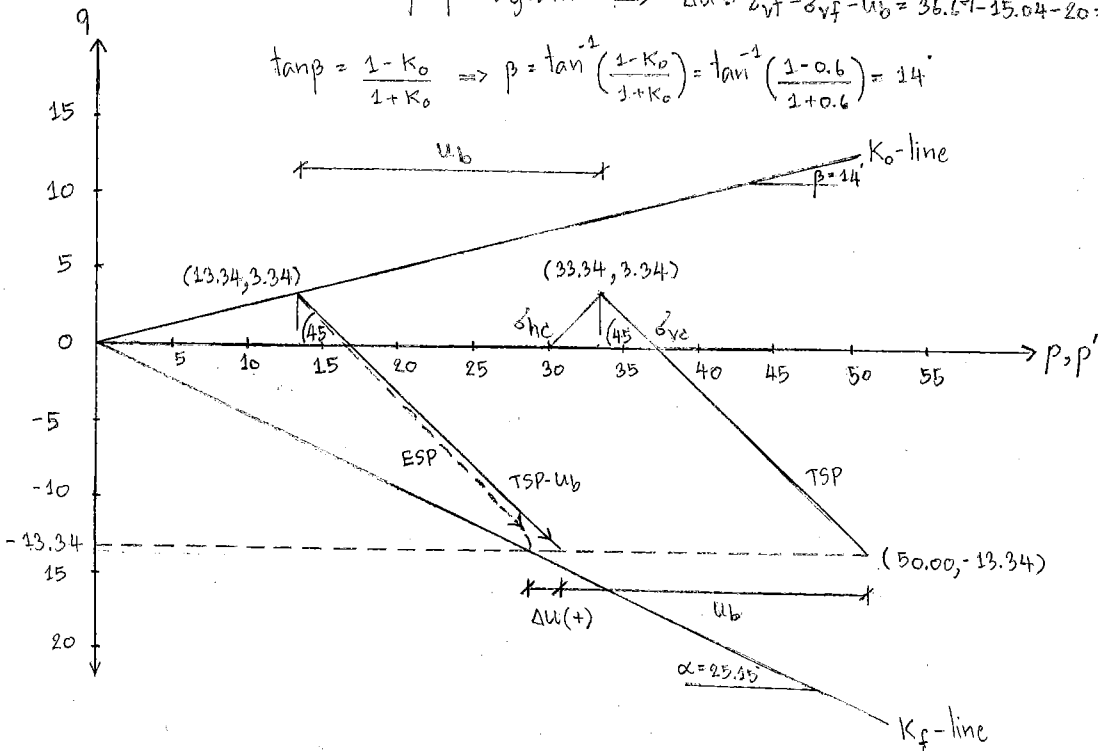
$$= 63.33 - 30 \quad \left| \quad \delta'_{vf} = 36.67 \text{ t/m}^2 \right.$$

$$= 33.33 \text{ t/m}^2 \quad \left| \quad \delta'_{hf} = 63.33 \text{ t/m}^2 \right.$$

$$\Delta u = A_f \times \Delta \sigma'_1 = 0.7 \times 33.33 = 23.33 \text{ t/m}^2$$

Assume Δu on p - q diagram $\implies \Delta u = \delta'_{vf} - \delta'_{vc} - u_b = 36.67 - 15.04 - 20 = 1.63 \text{ t/m}^2$

$$\tan \beta = \frac{1 - K_0}{1 + K_0} \implies \beta = \tan^{-1} \left(\frac{1 - K_0}{1 + K_0} \right) = \tan^{-1} \left(\frac{1 - 0.6}{1 + 0.6} \right) = 14^\circ$$



TSP on $(P_f, q_f) = (50.00, -13.34)$

$$u \text{ at failure} = u_b + \Delta u = 20 + 1.63 = 21.63 \text{ t/m}^2$$

Exam 1 Consolidation test on normally consolidated clay ($\sigma'_s = 0.3 \text{ t/m}^2$, $\sigma_v = 0.5 \text{ t/m}^2$, $K_0 = 1.5$ for initial condition) until $\sigma'_{vf} = 5 \text{ t/m}^2$, $K_{0 \text{ NC}} = 0.6$ for final condition

Solⁿ 1.) Initial condition before consolidated

$$\begin{aligned} \sigma_s &= \sigma'_s + u_s = 0 \\ u_s &= -\sigma'_s = -0.3 \text{ t/m}^2 \\ \sigma_{v0} &= 0.5 \text{ t/m}^2 \\ \sigma'_{v0} &= \sigma_{v0} - u_s = 0.5 - (-0.3) = 0.8 \text{ t/m}^2 \\ K_0 &= \frac{\sigma'_{h0}}{\sigma'_{v0}} \\ \sigma'_{h0} &= K_0 \sigma'_{v0} = 1.5 \times 0.8 = 1.2 \text{ t/m}^2 \\ \sigma_{h0} &= \sigma'_{h0} + u_s = 1.2 + (-0.3) = 0.9 \text{ t/m}^2 \end{aligned}$$

$$\begin{aligned} p &= \frac{\sigma_{v0} + \sigma_{h0}}{2} = \frac{0.5 + 0.9}{2} = 0.7 \\ q &= \frac{\sigma_{v0} - \sigma_{h0}}{2} = \frac{0.5 - 0.9}{2} = -0.2 \\ p' &= \frac{\sigma'_{v0} + \sigma'_{h0}}{2} = \frac{0.8 + 1.2}{2} = 1.0 \\ q' &= q = \frac{\sigma'_{v0} - \sigma'_{h0}}{2} = \frac{0.8 - 1.2}{2} = -0.2 \end{aligned}$$

ตัวอย่างดิน ส.ฉ.1 การทดสอบด้วย $-u_s$ คือการใส่ load ด้วย $\frac{\Delta p}{p} = 1.0$ เพราะ consolidated with $K_{0 \text{ NC}}$

$$\begin{aligned} \frac{\Delta p}{p} = 1.0 &\Rightarrow \Delta p = \sigma_{v1} = 1.0 \times p \\ \sigma_{v1} &= 1.0 \times \sigma_{v0} = 1.0 \times 0.5 = 0.5 \text{ t/m}^2 \end{aligned}$$

ตัวอย่างดิน ส.ฉ.1 σ'_s คงที่ ใส่ load ที่ทำให้ดิน saturated และ pore pressure เกิดขึ้นเล็กน้อย ($\Delta u < 0$) ดังนั้น พอจะเรียก consolidated คือการใส่ initial total stress และ effective stress ที่ minimum โดย

$$\begin{aligned} \sigma_{v1} &= \sigma'_{v1} = 0.5 \text{ t/m}^2 \\ \sigma_{h1} &= \sigma'_{h1} = K_0 \sigma'_{v1} = 1.5 \times 0.5 = 0.75 \text{ t/m}^2 \end{aligned}$$

$$\begin{aligned} p &= \frac{\sigma_{v1} + \sigma_{h1}}{2} = \frac{0.5 + 0.75}{2} = 0.625 \\ q &= \frac{\sigma_{v1} - \sigma_{h1}}{2} = \frac{0.5 - 0.75}{2} = -0.125 \\ p' &= \frac{\sigma'_{v1} + \sigma'_{h1}}{2} = \frac{0.5 + 0.75}{2} = 0.625 \\ q' &= \frac{\sigma'_{v1} - \sigma'_{h1}}{2} = \frac{0.5 - 0.75}{2} = -0.125 \end{aligned}$$

ใส่ load ด้วย $\Delta \sigma_v$ ที่ขึ้น

$$\begin{aligned} \text{load ที่เพิ่มขึ้น } \Delta \sigma_v &= \sigma_{vf} - \sigma_{v1} = 5 - 0.5 = 4.5 \text{ t/m}^2 \\ \text{การเพิ่มขึ้นของ } \Delta u & \text{ เกิดขึ้นในกรณี (เรียกว่า 4-D) } \Delta u = \Delta \sigma_v = 4.5 \text{ t/m}^2 \end{aligned}$$

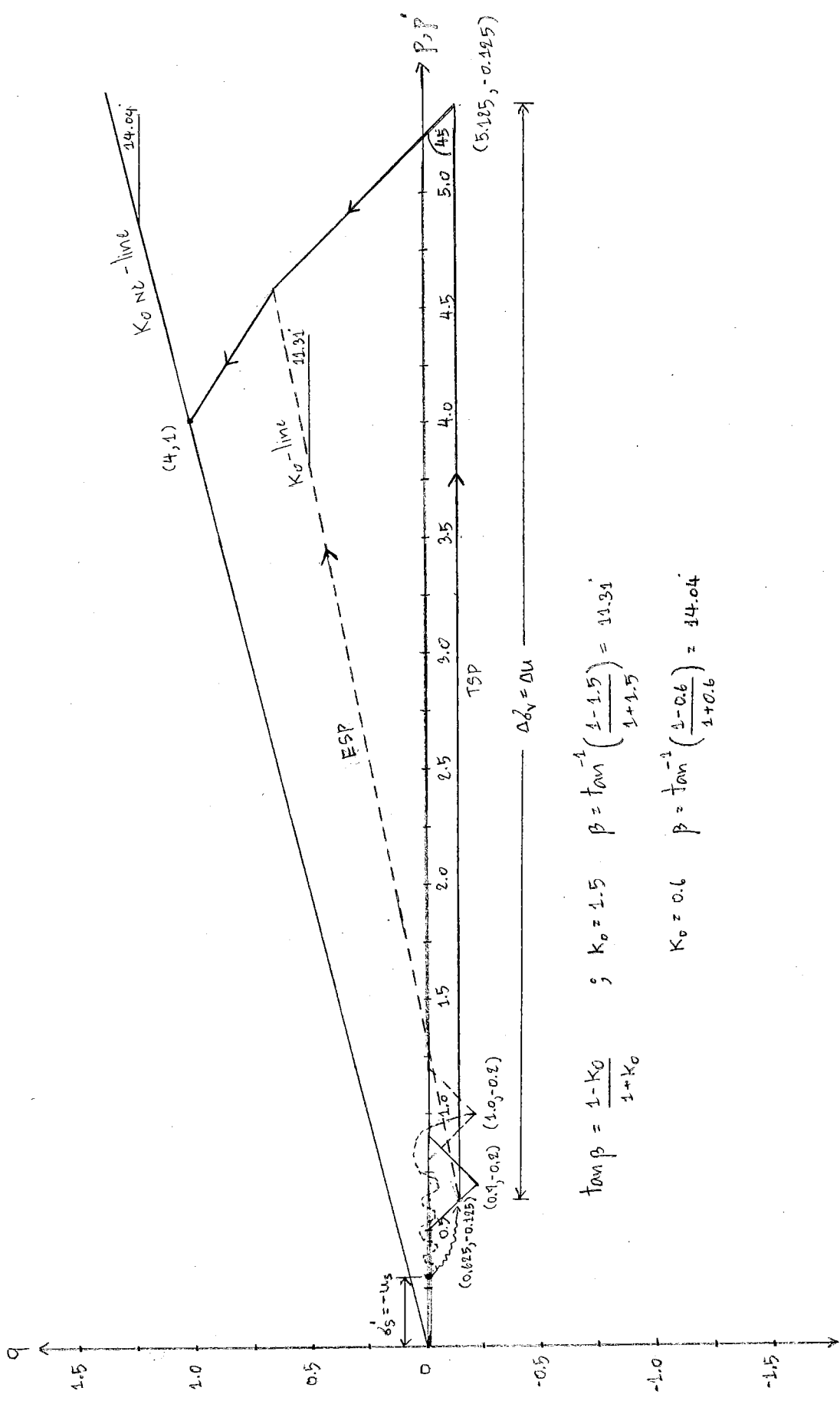
$$\begin{aligned} \sigma_{v2} &= \sigma_{v1} + \Delta \sigma_v = 0.5 + 4.5 = 5.00 \text{ t/m}^2 \\ \sigma_{h2} &= \sigma_{h1} + \Delta \sigma_v = 0.75 + 4.5 = 5.25 \text{ t/m}^2 \\ p &= \frac{\sigma_{v2} + \sigma_{h2}}{2} = \frac{5.00 + 5.25}{2} = 5.125 \\ q &= \frac{\sigma_{v2} - \sigma_{h2}}{2} = \frac{5.00 - 5.25}{2} = -0.125 \end{aligned}$$

ที่ final consolidation Δu ลดลงจาก $4.5 \text{ t/m}^2 \rightarrow 0$

กรณี TSP ให้ถือว่าที่เส้น $K_0 = 1.5$

ที่ failure TSP และ ESP ให้ใช้เส้นที่เดียวกัน หลังจากสิ้นสุด consolidation

$$\begin{aligned} \sigma_{vf} &= \sigma'_{vf} = 5 \text{ t/m}^2 \\ \sigma'_{hf} &= K_0 \sigma'_{vf} = 0.6 \times 5 = 3 \text{ t/m}^2 = \sigma_{hf} \\ p &= p' = \frac{\sigma_{vf} + \sigma_{hf}}{2} = \frac{\sigma'_{vf} + \sigma'_{hf}}{2} = \frac{5 + 3}{2} = 4 \\ q &= q' = \frac{\sigma_{vf} - \sigma_{hf}}{2} = \frac{\sigma'_{vf} - \sigma'_{hf}}{2} = \frac{5 - 3}{2} = 1 \end{aligned}$$



$$\tan \beta = \frac{1 - K_0}{1 + K_0} \quad ; \quad K_0 = 1.5 \quad \beta = \tan^{-1} \left(\frac{1 - 1.5}{1 + 1.5} \right) = 11.31^\circ$$

$$K_0 = 0.6 \quad \beta = \tan^{-1} \left(\frac{1 - 0.6}{1 + 0.6} \right) = 14.04^\circ$$

Exam 2 K_0 consolidated triaxial extension (unloading) test.

Given $\sigma'_c = 3 \text{ t/m}^2$, $K_0 = 0.6$, $u_b = 0 \text{ t/m}^2$ for initial condition

$\phi' = 29^\circ$, $c' = 0$, $q_f = 1.35 \text{ t/m}^2$ for failure condition

Solⁿ 1.) $\delta'_{hc} = \delta'_c = 3 \text{ t/m}^2$

$\delta_{hc} = \delta'_{hc} + u_b = 3 + 0 = 3 \text{ t/m}^2$

$K_0 = \frac{\delta'_{hc}}{\delta'_{vc}}$

$\delta'_{vc} = \frac{\delta'_{hc}}{K_0} = \frac{3}{0.6} = 5 \text{ t/m}^2$

$\delta_{vc} = \delta'_{vc} + u_b = 5 + 0 = 5 \text{ t/m}^2$

$p = \frac{\delta_{vc} + \delta_{hc}}{2} = \frac{5+3}{2} = 4$

$q = \frac{\delta_{vc} - \delta_{hc}}{2} = \frac{5-3}{2} = 1$

$p' = \frac{\delta'_{vc} + \delta'_{hc}}{2} = \frac{5+3}{2} = 4$

$q' = \frac{\delta'_{vc} - \delta'_{hc}}{2} = \frac{5-3}{2} = 1$

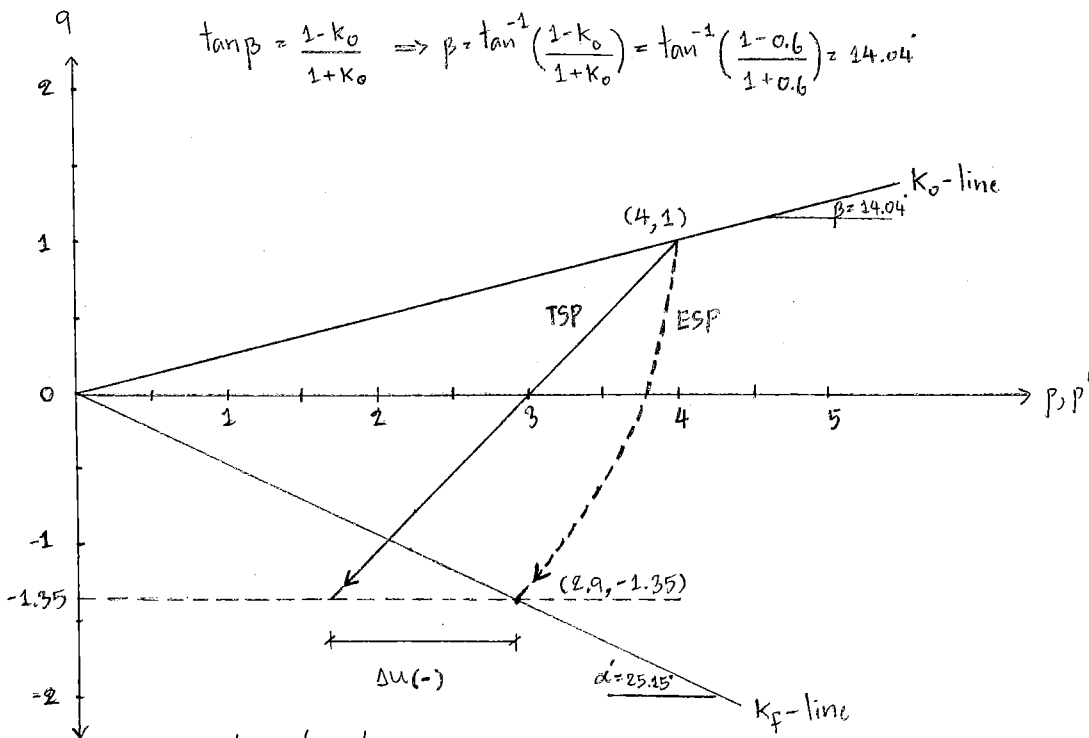
2.) at failure condition

$\tan \alpha' = \sin \phi' = \sin 29^\circ = 0.484$

$\alpha' = \tan^{-1}(\sin 29^\circ) = 25.15^\circ$

$a' = c' \cos \phi' = 0 \times \cos 29^\circ = 0$

$q_f = s_u = 1.35 \text{ t/m}^2$



$\tan \beta = \frac{1-K_0}{1+K_0} \Rightarrow \beta = \tan^{-1}\left(\frac{1-K_0}{1+K_0}\right) = \tan^{-1}\left(\frac{1-0.6}{1+0.6}\right) = 24.04^\circ$

$p'_f = \frac{\delta'_{vf} + \delta'_{hf}}{2} = 2.9$

$q'_f = \frac{\delta'_{vf} - \delta'_{hf}}{2} = -1.35$

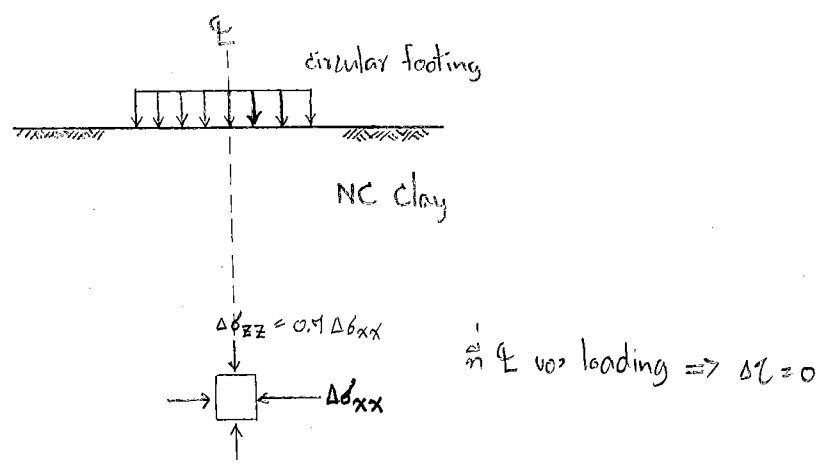
$\frac{\delta'_{2f}}{\delta'_{3f}} \sin \phi_f = \frac{4.25}{1.55} = 2.74$

(1) + (2) $\Rightarrow \delta'_{vf} = 2.9 - 1.35 = 1.55 \text{ t/m}^2 = \delta'_{2f}$

(1) - (2) $\Rightarrow \delta'_{hf} = 2.9 + 1.35 = 4.25 \text{ t/m}^2 = \delta'_{1f}$

Examin. 3 At a point underlying the circular footing for normally consolidated clay, the clay is loaded with $\Delta\sigma_{zz} = 0.1 \Delta\sigma_{xx}$. The clay has $K_0 = 0.6$, $\sigma'_{v0} = 5 \text{ t/m}^2$ and $\sigma'_{hf} = 8 \text{ t/m}^2$, $u_0 = 3 \text{ t/m}^2$ and $u_f = 6 \text{ t/m}^2$.

Solⁿ



► initial condition

$$\sigma'_{v0} = 5 \text{ t/m}^2, \quad \sigma_{v0} = \sigma'_{v0} + u_0 = 5 + 3 = 8 \text{ t/m}^2$$

$$\sigma'_{h0} = K_0 \sigma'_{v0} = 0.6 \times 5 = 3 \text{ t/m}^2, \quad \sigma_{h0} = \sigma'_{h0} + u_0 = 3 + 3 = 6 \text{ t/m}^2$$

$$p = \frac{\sigma_{v0} + \sigma_{h0}}{2} = \frac{8 + 6}{2} = 7$$

$$p' = \frac{\sigma'_{v0} + \sigma'_{h0}}{2} = \frac{5 + 3}{2} = 4$$

$$q = \frac{\sigma_{v0} - \sigma_{h0}}{2} = \frac{8 - 6}{2} = 1$$

$$q' = \frac{\sigma'_{v0} - \sigma'_{h0}}{2} = \frac{5 - 3}{2} = 1$$

► after loading

$$\sigma'_{vf} = 8 \text{ t/m}^2$$

$$\Delta\sigma_{zz} = \sigma_{vf} - \sigma_{v0} = [\sigma'_{vf} + (u_0 + \Delta u)] - \sigma_{v0} = [\sigma'_{vf} + u_f] - \sigma_{v0}$$

$$\Delta\sigma_{zz} = [8 + 6] - 8 = 6 \text{ t/m}^2$$

$$\sigma_{vf} = \sigma_{v0} + \Delta\sigma_{zz} = 8 + 6 = 14 \text{ t/m}^2$$

$$\Delta\sigma_{xx} = \frac{\Delta\sigma_{zz}}{0.1} = \frac{6}{0.1} = 60 \text{ t/m}^2$$

$$\sigma'_{hf} = \sigma'_{h0} + \Delta\sigma_{xx} = 3 + 60 = 63 \text{ t/m}^2$$

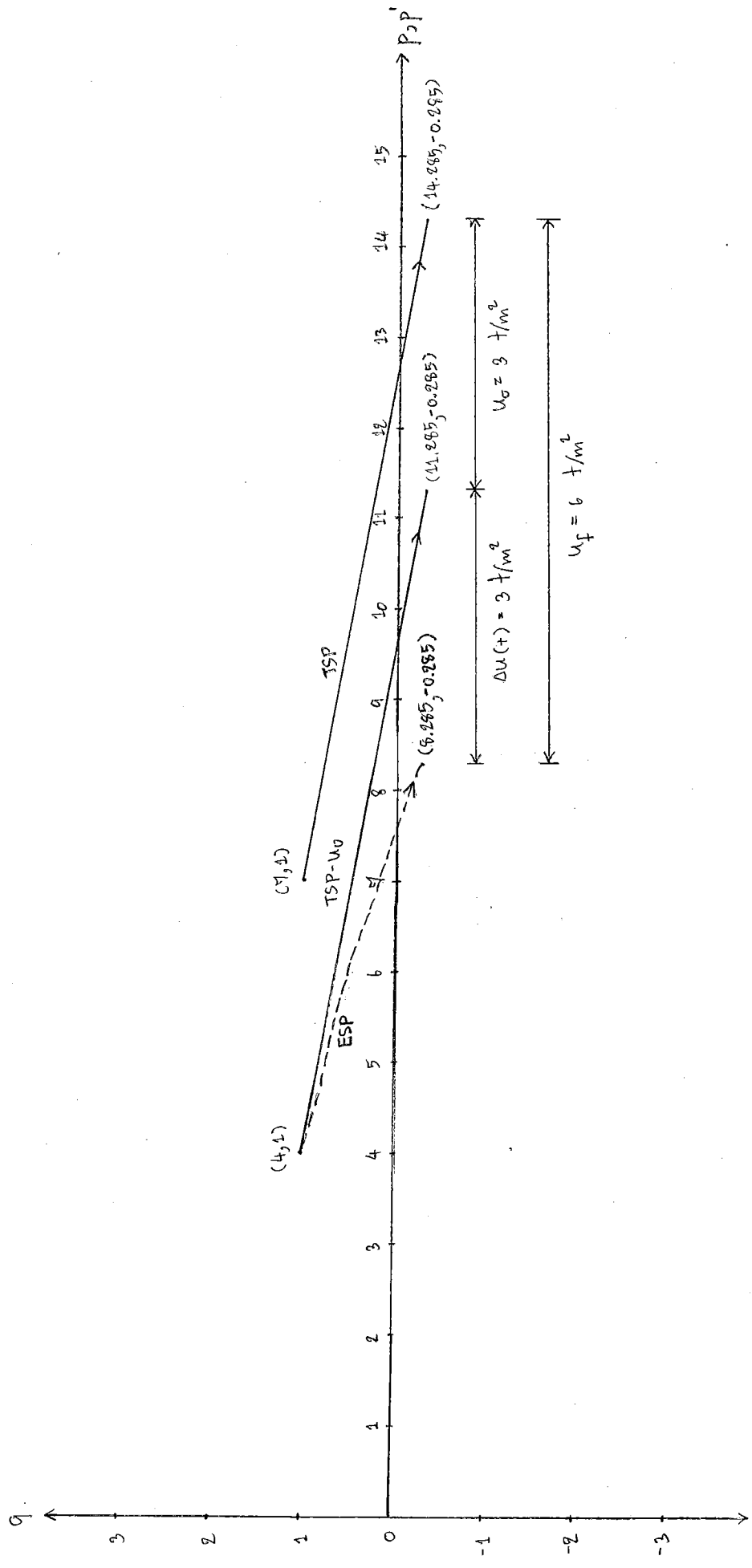
$$\sigma'_{hf} = (\sigma'_{h0} + \Delta\sigma_{xx}) - (u_0 + \Delta u) = (\sigma'_{h0} + \Delta\sigma_{xx}) - u_f = (3 + 60) - 6 = 57 \text{ t/m}^2$$

$$p_f = \frac{\sigma_{vf} + \sigma_{hf}}{2} = \frac{14 + 63}{2} = 38.5$$

$$p'_f = \frac{\sigma'_{vf} + \sigma'_{hf}}{2} = \frac{8 + 57}{2} = 32.5$$

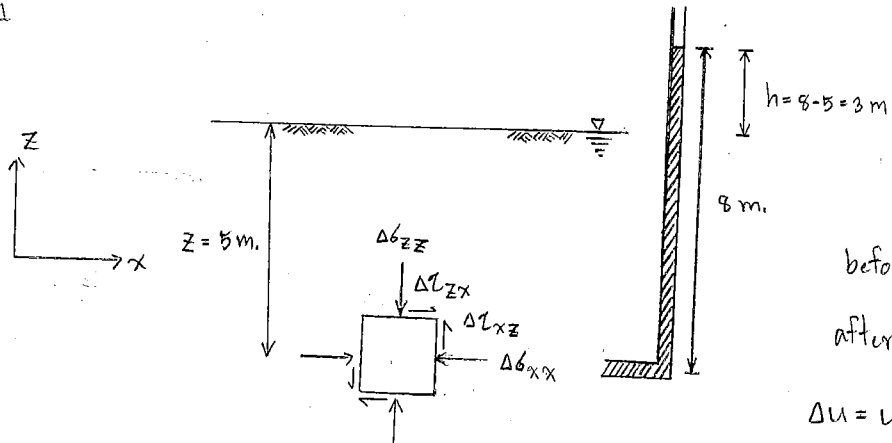
$$q_f = \frac{\sigma_{vf} - \sigma_{hf}}{2} = \frac{14 - 63}{2} = -24.5$$

$$q'_f = \frac{\sigma'_{vf} - \sigma'_{hf}}{2} = \frac{8 - 57}{2} = -24.5$$



Exam 4 The normally consolidated clay at the depth of 5 m. was stresses with $\Delta\sigma_{zz} = 4 \text{ t/m}^2$
 $\Delta\sigma_{xx} = 1.0 \text{ t/m}^2$ and $\Delta\tau_{xz} = 0.9 \text{ t/m}^2$. The piezometric head after the loading is
 8 m. Find the Skempton's pore pressure parameter, considered the ground water is
 at the ground surface.

Solⁿ



before loading $u_1 = \gamma_w \times z$

after loading $u_2 = \gamma_w \times (z+h)$

$$\Delta u = u_2 - u_1 = \gamma_w(z+h) - \gamma_w \times z = \gamma_w \times h$$

$$\Delta\sigma_{xx} = 1.0 \text{ t/m}^2, \Delta\sigma_{zz} = 4.0 \text{ t/m}^2 \quad (+\text{v.e.} \rightarrow \text{compression})$$

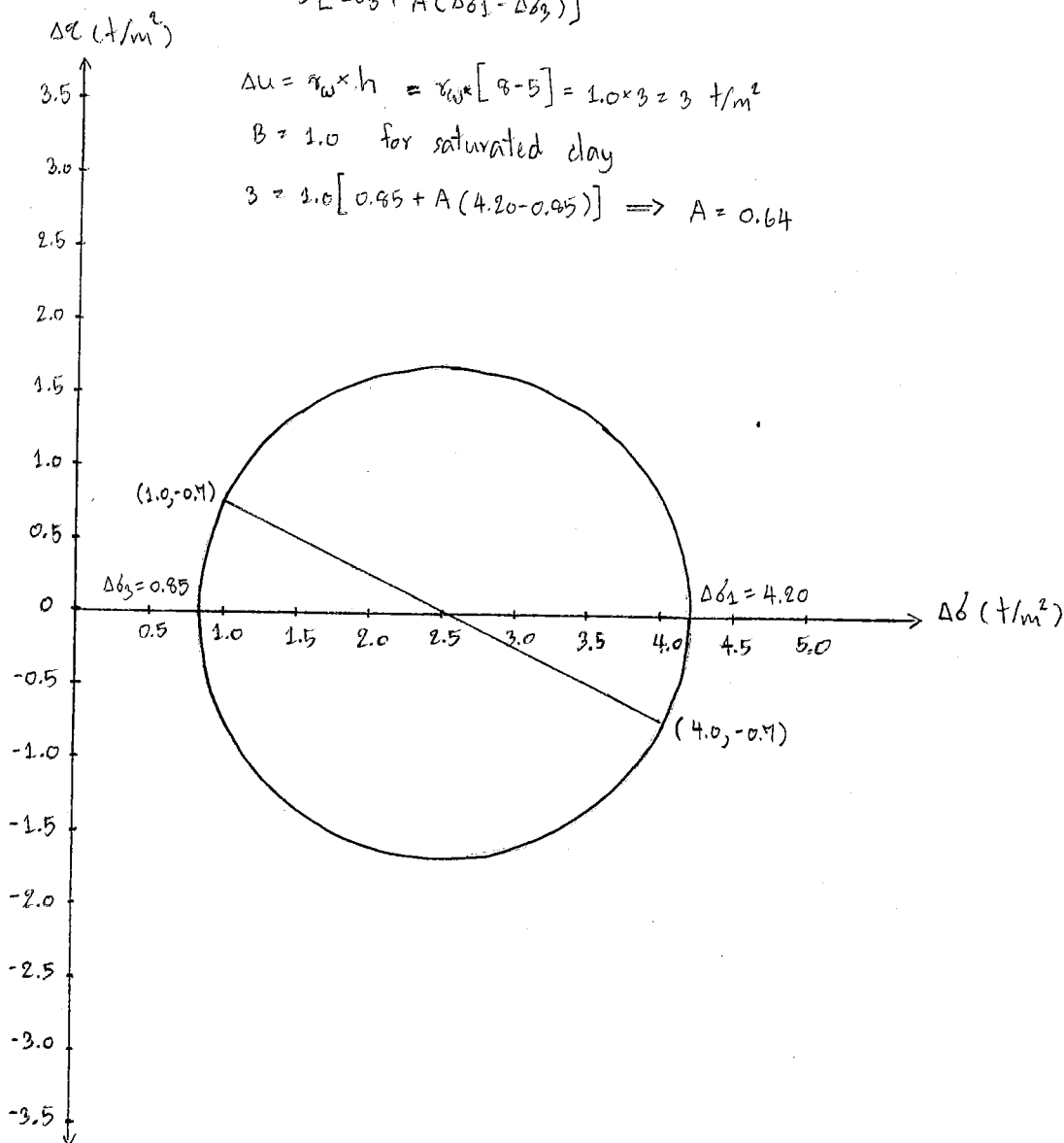
$$\Delta\tau_{xz} = 0.9 \text{ t/m}^2 \quad (+\text{v.c.} \rightarrow \text{counter clockwise})$$

$$\Delta u = B [\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$\Delta u = \gamma_w \times h = \gamma_w \times [8-5] = 1.0 \times 3 = 3 \text{ t/m}^2$$

$B = 1.0$ for saturated clay

$$3 = 1.0 [0.85 + A(4.20 - 0.85)] \Rightarrow A = 0.64$$

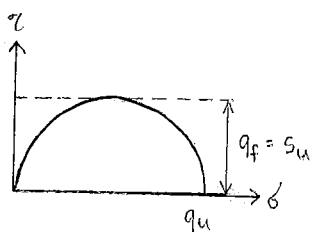


Question 1 Three perfectly undisturbed samples from three different depths were taken from a normally consolidated homogeneous clay layer. The unconfined compression test is carried out on those samples. The results are as shown in the table.

Location	Depth (m)	$(\sigma'_c)_{in-situ}$ (t/m^2)	S_u (t/m^2)
A	3	1.8	3.8
B	8	4.8	5.5
C	12	7.2	7.0

If a series of consolidated undrained triaxial tests are performed under three isotropic stresses (shown in the table) on samples taken from A, estimate the friction angle (ϕ') and cohesion (c'). Assume that the Skempton's pore pressure parameter at failure (A_f) is 0.9.

Solⁿ



from unconfined compression test

$$q_f = \frac{\sigma_{1f} - \sigma_{3f}}{2} = S_u$$

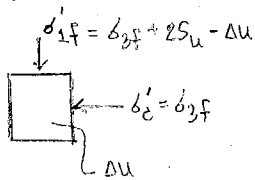
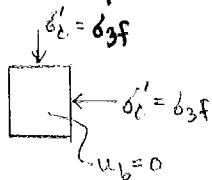
$$\sigma_{1f} = \sigma_{3f} + 2S_u, \quad \sigma'_{3f} = \sigma_c$$

Assume that soil samples were taken from the same depth in CU triaxial test as soil samples in UC test (has the same σ'_c) so that Mohr's circle of CU triaxial test has radius ($q_f = S_u$) as those samples in UC test.

CU triaxial test assume back pressure, $u_b = 0$ t/m^2

$$\sigma_c = \sigma'_c + u_b = \sigma'_c$$

CU triaxial compression loading test (σ'_c is σ_{3f})



$$\Delta \sigma_3 = 0, \quad \Delta \sigma_1 = \sigma_{1f} - \sigma_{3f}$$

$$\Delta u = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]$$

for saturated clay $\Rightarrow B = 1.0$, $\frac{1}{n}$ failure $\Rightarrow A_f = 0.9$

$$\Delta u = A_f \cdot \Delta \sigma_1$$

$$\sigma'_{3f} = \sigma_{3f} - \Delta u$$

$$\sigma'_{1f} = \sigma_{1f} - \Delta u$$

δ'_c (t/m ²)	δ_c (t/m ²)	S_u (t/m ²)	δ_{3f} (t/m ²)	δ_{1f} (t/m ²)	$\Delta\delta_1$ (t/m ²)	Δu (t/m ²)	δ'_{3f} (t/m ²)	δ'_{1f} (t/m ²)
1.8	1.8	3.8	1.8	9.4	7.6	5.32	-3.52	4.08
4.8	4.8	5.5	4.8	15.8	11.0	7.70	-2.90	8.10
7.2	7.2	7.0	7.2	21.2	14.0	9.80	-2.60	11.40

$$\delta_c = \delta'_c + \overset{0}{\uparrow} \delta_b = \delta'_c$$

$$\delta_{3f} = \delta_c$$

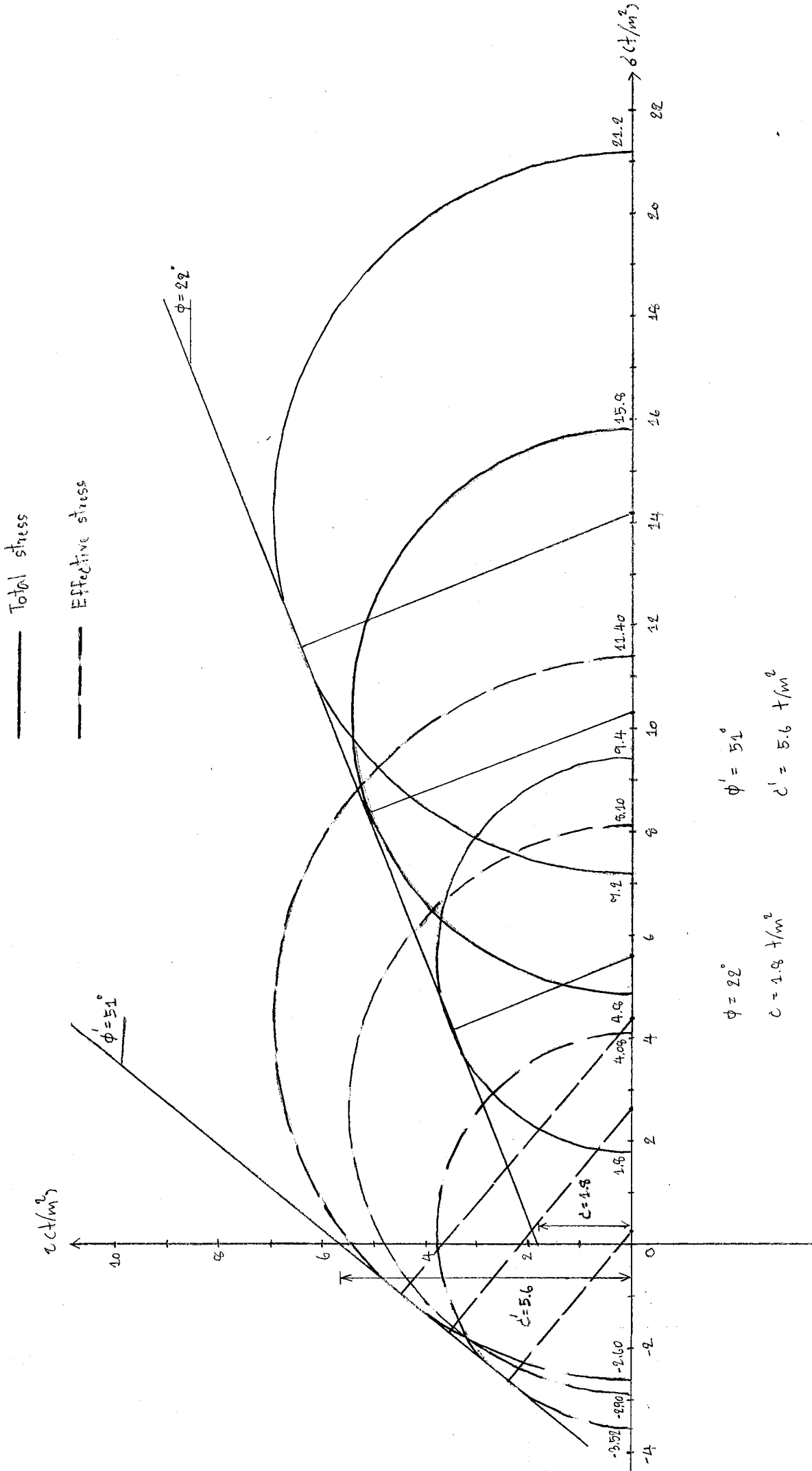
$$\delta_{1f} = \delta_{3f} + 2S_u$$

$$\Delta\delta_1 = \delta_{1f} - \delta_{3f}$$

$$\Delta u = A_f \times \Delta\delta_1 = 0.7 \times \Delta\delta_1$$

$$\delta'_{3f} = \delta_{3f} - \Delta u$$

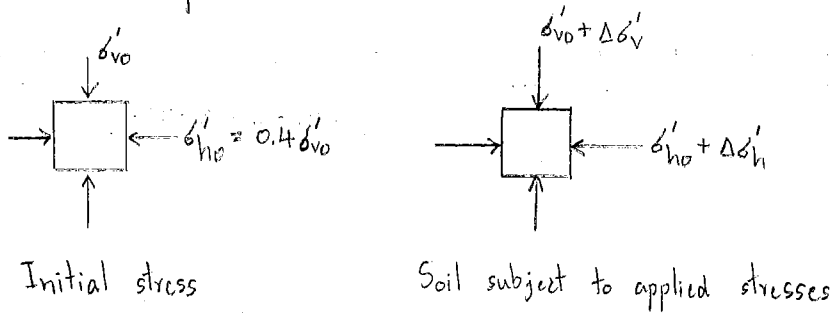
$$\delta'_{1f} = \delta_{1f} - \Delta u$$



$\phi' = 51^\circ$
 $c' = 5.6 \text{ t/m}^2$
 $\phi = 22^\circ$
 $c = 1.8 \text{ t/m}^2$

Question 2 A soil obeys the Mohr-Coulomb failure criterion, in which the effective stress parameters are: $\alpha' = 0.4 \frac{1}{\sqrt{3}}$, $\beta' = 2\alpha'$. The soil begins with an anisotropic state of stress as shown in the figure below. The incremental effective stresses are applied such that $\Delta\sigma'_h = -\frac{\Delta\sigma'_v}{4}$, where $\Delta\sigma'_v > 0$ until the soil fails.

- 1.) Draw the stress path in p-q diagram and mark the point of failure on K_f line
- 2.) Calculate the shear and normal stresses on the failure plane and the orientation of the failure plane.



Solⁿ 1.) Initial condition

$$p' = \frac{\sigma'_{v0} + \sigma'_{ho}}{2} = \frac{\sigma'_{v0} + 0.4\sigma'_{v0}}{2} = 0.7\sigma'_{v0}$$

$$q = \frac{\sigma'_{v0} - \sigma'_{ho}}{2} = \frac{\sigma'_{v0} - 0.4\sigma'_{v0}}{2} = 0.3\sigma'_{v0}$$

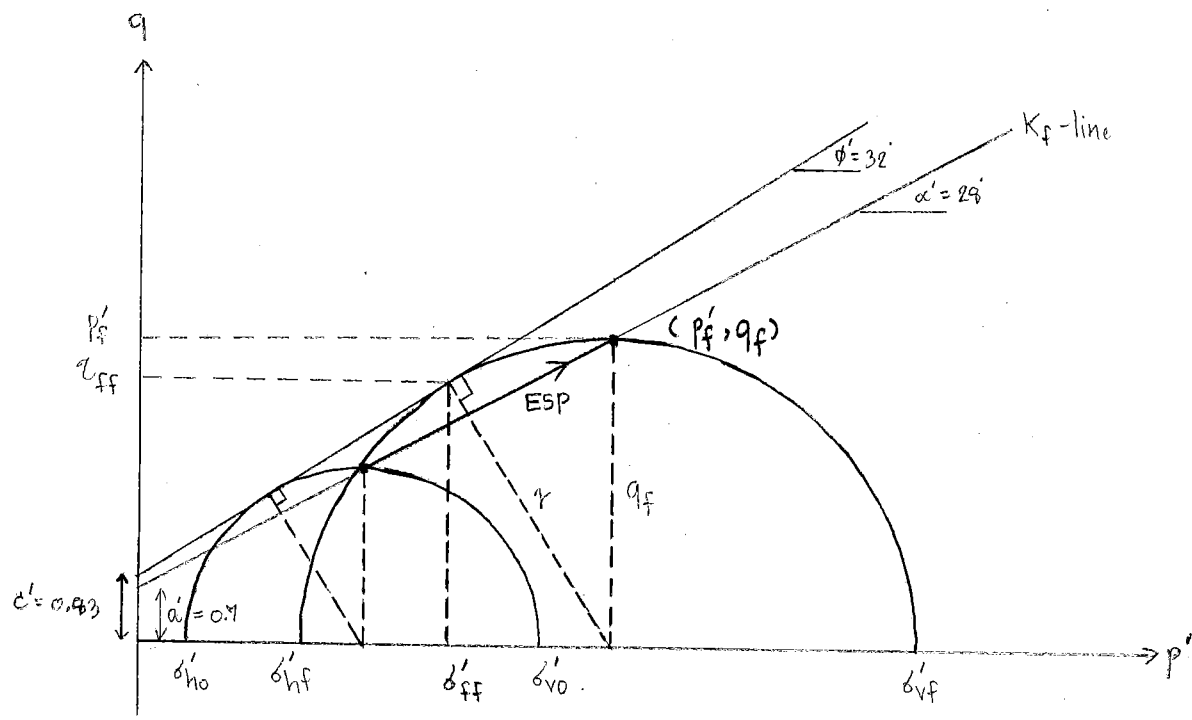
2.) After loading

$$\sigma'_{vf} = \sigma'_{v0} + \Delta\sigma'_v$$

$$\sigma'_{hf} = \sigma'_{ho} + \Delta\sigma'_h = 0.4\sigma'_{v0} + \left(-\frac{\Delta\sigma'_v}{4}\right) = 0.4\sigma'_{v0} - 0.25\Delta\sigma'_v$$

$$p'_f = \frac{\sigma'_{vf} + \sigma'_{hf}}{2} = \frac{(\sigma'_{v0} + \Delta\sigma'_v) + (0.4\sigma'_{v0} - 0.25\Delta\sigma'_v)}{2} = 0.7\sigma'_{v0} + 0.375\Delta\sigma'_v$$

$$q'_f = \frac{\sigma'_{vf} - \sigma'_{hf}}{2} = \frac{(\sigma'_{v0} + \Delta\sigma'_v) - (0.4\sigma'_{v0} - 0.25\Delta\sigma'_v)}{2} = 0.3\sigma'_{v0} + 0.625\Delta\sigma'_v$$



$$\sin \phi' = \tan \alpha' \implies \phi' = \sin^{-1}(\tan \alpha') = \sin^{-1}(\tan 28^\circ) = 32^\circ$$

$$c' = \frac{a'}{\cos \phi'} = \frac{0.7}{\cos 32^\circ} = 0.83 \text{ t/m}^2$$

210 $q_f = p'_f \tan \alpha' + a$

$$0.36 \delta'_{vo} + 0.625 \Delta \delta'_v = (0.7 \delta'_{vo} + 0.345 \Delta \delta'_v) \tan 28^\circ + 0.7$$

$$(0.625 - 0.345 \tan 28^\circ) \Delta \delta'_v = (0.7 \tan 28^\circ - 0.3) \delta'_{vo} + 0.7$$

$$\Delta \delta'_v = \frac{(0.7 \tan 28^\circ - 0.3) \delta'_{vo} + 0.7}{(0.625 - 0.345 \tan 28^\circ)}$$

$$\Delta \delta'_v = 0.169 \delta'_{vo} + 1.64$$

111411 $p'_f = 0.7 \delta'_{vo} + 0.345 \times \Delta \delta'_v = 0.7 \delta'_{vo} + 0.345 \times (0.169 \delta'_{vo} + 1.64) = 0.5163 \delta'_{vo} + 0.615$

$$q_f = 0.36 \delta'_{vo} + 0.625 \times \Delta \delta'_v = 0.36 \delta'_{vo} + 0.625 \times (0.169 \delta'_{vo} + 1.64) = 0.406 \delta'_{vo} + 1.025$$

2111 p'-q diagram

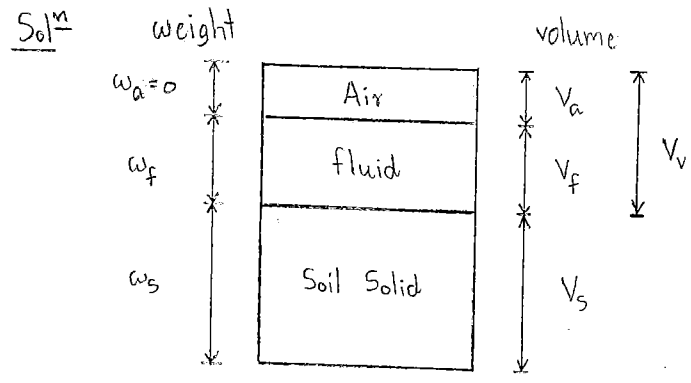
$$\delta'_{ff} = p'_f - r \sin \phi' = p'_f - q_f \sin \phi' = (0.5163 \delta'_{vo} + 0.615) - (0.406 \delta'_{vo} + 1.025) \sin 32^\circ = 0.547 \delta'_{vo} + 0.072$$

$$z_{ff} = r \cos \phi' = q_f \cos \phi' = (0.406 \delta'_{vo} + 1.025) \cos 32^\circ = 0.344 \delta'_{vo} + 0.869$$

Question 3 For a soil having a fluid replaces water in voids, specific gravity of fluid is G_f and its weight is w_f .

Determine 1.) Degree of saturation, S

2.) Air-voids content, A_v



$$1.) e = \frac{V_v}{V_s} = \frac{\frac{V_f}{S}}{\frac{w_s}{\gamma_s}} = \frac{\frac{w_f}{\gamma_f \cdot S}}{\frac{w_s}{\gamma_s}} = \frac{w_f}{w_s} \times \frac{\gamma_s}{\gamma_f} \times \frac{1}{S}$$

$$S = \frac{w_f}{w_s} \times \frac{\gamma_s}{\gamma_f} \times \frac{1}{e} = \frac{w_f}{w_s} \times \frac{\gamma_s}{G_f \cdot \gamma_w} \times \frac{1}{e} = \frac{w_f}{w_s} \times \frac{G_s}{G_f} \times \frac{1}{e}$$

$$2.) V_a = V_v - V_f = V_v - S \cdot V_v = (1-S)V_v = (1-S) \cdot e \cdot V_s$$

since $e = \frac{V_v}{V_s}$

$$e+1 = \frac{V_v}{V_s} + 1 = \frac{V_v + V_s}{V_s} = \frac{V}{V_s}$$

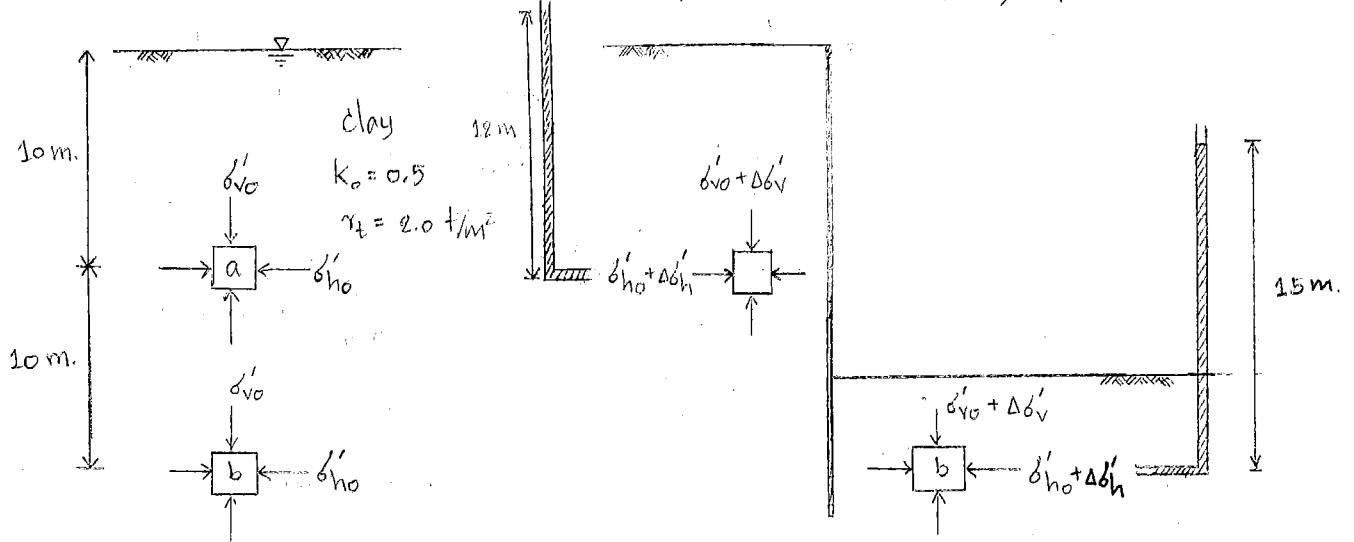
$$\frac{V_s}{V} = \frac{1}{1+e}$$

$$\frac{V_a}{V} = (1-S) \cdot e \cdot \frac{V_s}{V} = (1-S) \cdot e \cdot \frac{1}{1+e}$$

$$A_v = \frac{V_a}{V} = (1-S) \cdot \frac{e}{1+e}$$

Question 4 Clay obeys the Mohr-Coulomb failure criterion, which has initial state of stress as shown in the figure below. The horizontal stress reduced by excavation which is dug at depth from ground surface in a few days.
 Draw p-q diagram of soil at a, b in short term and long term.

Given for a $\Delta\sigma'_h = -4 \text{ t/m}^2$, $\Delta\sigma'_v = -3 \text{ t/m}^2$; for b $\Delta\sigma'_h = -8 \text{ t/m}^2$, $\Delta\sigma'_v = -15 \text{ t/m}^2$



Solⁿ 1.) soil element A

► Initial stress $\sigma'_{vo} = \gamma_t \cdot z = 2.0 \times 10 = 20 \text{ t/m}^2$, $u_0 = \gamma_w \cdot z = 1.0 \times 10 = 10 \text{ t/m}^2$

$$\sigma'_{vo} = \sigma_{vo} - u_0 = 20 - 10 = 10 \text{ t/m}^2$$

$$\sigma'_{ho} = K_0 \cdot \sigma'_{vo} = 0.5 \times 10 = 5 \text{ t/m}^2$$

$$\sigma_{ho} = \sigma'_{ho} + u_0 = 5 + 10 = 15 \text{ t/m}^2$$

$$p_0 = \frac{\sigma_{vo} + \sigma_{ho}}{2} = \frac{20 + 15}{2} = 17.5$$

$$q_0 = \frac{\sigma_{vo} - \sigma_{ho}}{2} = \frac{20 - 15}{2} = 2.5$$

$$p'_0 = \frac{\sigma'_{vo} + \sigma'_{ho}}{2} = \frac{10 + 5}{2} = 7.5$$

$$q'_0 = \frac{\sigma'_{vo} - \sigma'_{ho}}{2} = \frac{10 - 5}{2} = 2.5$$

► End of construction $\sigma_{vu} = \sigma_{vo} + \Delta\sigma'_v = 20 - 3 = 17 \text{ t/m}^2$ before excavation $u_1 = \gamma_w \cdot z$
 $\sigma'_{vu} = \sigma'_{vo} + \Delta\sigma'_v = \sigma'_{vo} + \Delta\sigma'_v - \Delta u = 10 - 3 - 2 = 5 \text{ t/m}^2$ after excavation $u_2 = \gamma_w \cdot h$
 $\sigma_{hu} = \sigma_{ho} + \Delta\sigma'_h = 15 - 4 = 11 \text{ t/m}^2$ $\Delta u = u_2 - u_1 = \gamma_w \cdot (h - z)$
 $\sigma'_{hu} = \sigma'_{ho} + \Delta\sigma'_h = \sigma'_{ho} + \Delta\sigma'_h - \Delta u = 5 - 4 - 2 = -1 \text{ t/m}^2$ $= 1.0 \times (12 - 10) = 2 \text{ t/m}^2$

$$p = \frac{\sigma_{vu} + \sigma_{hu}}{2} = \frac{17 + 11}{2} = 14$$

$$q = \frac{\sigma_{vu} - \sigma_{hu}}{2} = \frac{17 - 11}{2} = 3$$

$$p' = \frac{\sigma'_{vu} + \sigma'_{hu}}{2} = \frac{5 + (-1)}{2} = 2$$

$$q' = \frac{\sigma'_{vu} - \sigma'_{hu}}{2} = \frac{5 - (-1)}{2} = 3$$

► End of consolidation (Long term)

$\Delta u = 0$

$\delta v_f = \delta v_0 + \Delta \delta v = 20 - 3 = 17 \text{ t/m}^2$

$\delta' v_f = \delta' v_0 + \Delta \delta v - \delta u^0 = 10 - 3 = 7 \text{ t/m}^2$

$\delta h_f = \delta h_0 + \Delta \delta h = 15 - 4 = 11 \text{ t/m}^2$

$\delta' h_f = \delta' h_0 + \Delta \delta h - \delta u^0 = 5 - 4 = 1 \text{ t/m}^2$

$p_f = \frac{\delta v_f + \delta h_f}{2} = \frac{17 + 11}{2} = 14$

$q_f = \frac{\delta v_f - \delta h_f}{2} = \frac{17 - 11}{2} = 3$

$p'_f = \frac{\delta' v_f + \delta' h_f}{2} = \frac{7 + 1}{2} = 4$

$q'_f = \frac{\delta' v_f - \delta' h_f}{2} = \frac{7 - 1}{2} = 3$

2.) soil element B

► Initial stress $\delta v_0 = \gamma_s \cdot z = 2.0 \cdot 20 = 40 \text{ t/m}^2$, $u_0 = \gamma_w \cdot z = 1.0 \cdot 20 = 20 \text{ t/m}^2$

$\delta' v_0 = \delta v_0 - u_0 = 40 - 20 = 20 \text{ t/m}^2$

$\delta' h_0 = K_0 \cdot \delta' v_0 = 0.5 \cdot 20 = 10 \text{ t/m}^2$

$\delta h_0 = \delta' h_0 + u_0 = 10 + 20 = 30 \text{ t/m}^2$

$p_0 = \frac{\delta v_0 + \delta h_0}{2} = \frac{40 + 30}{2} = 35$

$q_0 = \frac{\delta v_0 - \delta h_0}{2} = \frac{40 - 30}{2} = 5$

$p'_0 = \frac{\delta' v_0 + \delta' h_0}{2} = \frac{20 + 10}{2} = 15$

$q'_0 = \frac{\delta' v_0 - \delta' h_0}{2} = \frac{20 - 10}{2} = 5$

► End of construction (Short term)

before excavation $u_1 = \gamma_w \cdot z$

after excavation $u_2 = \gamma_w \cdot h$

$\Delta u = u_2 - u_1 = \gamma_w \cdot (h - z) = 1.0 \cdot (15 - 20) = -5 \text{ t/m}^2$

$\delta v_u = \delta v_0 + \Delta \delta v = 40 - 15 = 25 \text{ t/m}^2$

$\delta' v_u = \delta' v_0 + \Delta \delta v = \delta' v_0 + \Delta \delta v - \Delta u = 20 + (-15) - (-5) = 10 \text{ t/m}^2$

$\delta h_u = \delta h_0 + \Delta \delta h = 30 - 8 = 22 \text{ t/m}^2$

$\delta' h_u = \delta' h_0 + \Delta \delta h = \delta' h_0 + \Delta \delta h - \Delta u = 10 + (-8) - (-5) = 7 \text{ t/m}^2$

$p = \frac{\delta v_u + \delta h_u}{2} = \frac{25 + 22}{2} = 23.5$

$q = \frac{\delta v_u - \delta h_u}{2} = \frac{25 - 22}{2} = 1.5$

$p' = \frac{\delta' v_u + \delta' h_u}{2} = \frac{10 + 7}{2} = 8.5$

$q' = \frac{\delta' v_u - \delta' h_u}{2} = \frac{10 - 7}{2} = 1.5$

► End of consolidation (Long term)

$$\Delta u = 0$$

$$\sigma_{vf} = \sigma_{vo} + \Delta\sigma_v = 40 + (-15) = 25 \text{ t/m}^2$$

$$\sigma'_{vf} = \sigma'_{vo} + \Delta\sigma'_v = \sigma'_{vo} + \Delta\sigma_v - \Delta u = 20 + (-15) = 5 \text{ t/m}^2$$

$$\sigma_{hf} = \sigma_{ho} + \Delta\sigma_h = 30 + (-8) = 22 \text{ t/m}^2$$

$$\sigma'_{hf} = \sigma'_{ho} + \Delta\sigma'_h = \sigma'_{ho} + \Delta\sigma_h - \Delta u = 10 + (-8) = 2 \text{ t/m}^2$$

$$p_f = \frac{\sigma_{vf} + \sigma_{hf}}{2} = \frac{25 + 22}{2} = 23.5$$

$$q_f = \frac{\sigma_{vf} - \sigma_{hf}}{2} = \frac{25 - 22}{2} = 1.5$$

$$p'_f = \frac{\sigma'_{vf} + \sigma'_{hf}}{2} = \frac{5 + 2}{2} = 3.5$$

$$q'_f = \frac{\sigma'_{vf} - \sigma'_{hf}}{2} = \frac{5 - 2}{2} = 1.5$$

