หลักสูตร การออกแบบโครงสร้างอาคารสูง

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โดย

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ร่วมกับ

ฝ่ายการศึกษาต่อเนื่อง จุฬาฯ

Chapter V

Conventional Beam - Slab System

A) Structural framing

- Secondary beams
- Main beams
- Tributary area

B) One way floor slabs

- Effective span, Slab thickness
- Bending moment, Load Pattern, Moment redistribution
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c) Waffle slab

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- Slab supported on three sides
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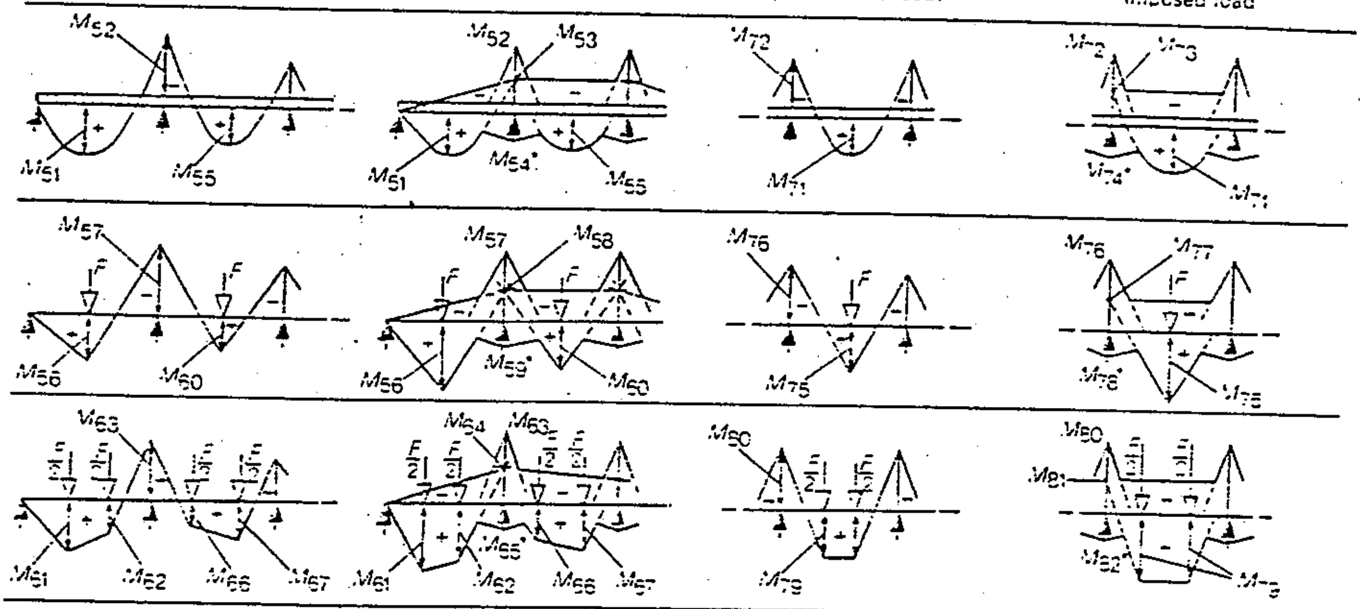
Continuous beams: Bending moment diagrams: Four or more equal spans

End span and penultimate span and support of infinite system		Internal span and support of infinite system	
Dead load (all spans loaded)	Imposed load	Dead load (all spans loaded)	Imposed load



Equal total load F on each loaded span
Bending moment = coefficient × F × span
Diagrams are symmetrical but are not drawn to scale

Moments indicated thus * do not result from loading arrangement prescribed in Code, which gives zero positive moment at all supports. Values indicated thus ( ) give maximum percentage reduction of span moment due to imposed load possible when support moments have already been reduced by full 30%.

System	Loading	Moment	Dead load (all spans loaded) nil	CP 110 10%	CP 110 30%	CP 114 15%	Imposed load nil	CP 110 10%	CP 110 30%	CP 114 15%
End span, and penultimate support and span of infinite system	Uniform loads	M_{51}	−0.073	−0.082	−0.091	−0.084	−0.100	−0.090	−0.068 (12)	−0.096
		M_{52}	−0.106	−0.095	−0.074	−0.090	−0.115	−0.104	−0.081	−0.099
		M_{53}	—	—	—	—	−0.054	−0.076	−0.081	−0.062
		M_{54}*	—	—	—	—	−0.014	−0.013	−0.010	−0.012
		M_{55}	−0.034	−0.043	−0.061	−0.047	−0.079	−0.071	−0.056 (20)	−0.071
	Central point loads	M_{56}	−0.171	−0.178	−0.194	−0.182	−0.210	−0.189	−0.189 (10)	−0.203
		M_{57}	−0.159	−0.143	−0.111	−0.135	−0.174	−0.157	−0.122	−0.148
		M_{58}	—	—	—	—	−0.079	−0.122	−0.122	−0.093
		M_{59}*	—	—	—	—	−0.021	−0.019	−0.015	−0.018
		M_{60}	−0.113	−0.127	−0.154	−0.133	−0.181	−0.163	−0.145 (30)	−0.171
	Third-point loads	M_{61}	−0.120	−0.124	−0.134	−0.127	−0.143	−0.129	−0.130 (9)	−0.139
		M_{62}	−0.072	−0.082	−0.101	−0.087	−0.119	−0.107	−0.094 (21)	−0.111
		M_{63}	−0.141	−0.127	−0.099	−0.120	−0.155	−0.140	−0.109	−0.132
		M_{64}	—	—	—	—	−0.072	−0.114	−0.109	−0.083
		M_{65}*	—	—	—	—	−0.019	−0.017	−0.013	−0.016
		M_{66}	−0.036	−0.051	−0.077	−0.056	−0.103	−0.092	−0.072 (30)	−0.093
		M_{67}	−0.051	−0.062	−0.086	−0.065	−0.109	−0.098	−0.076 (30)	−0.100
Internal span and support of infinite system	Uniform loads	M_{71}	−0.042	−0.050	−0.067	−0.054	−0.083	−0.075	−0.058 (20)	−0.077
		M_{72}	−0.083	−0.075	−0.058	−0.071	−0.106	−0.095	−0.074	−0.090
		M_{73}	—	—	—	—	−0.042	−0.050	−0.067	−0.048
		M_{74}*	—	—	—	—	−0.023	−0.025	−0.020	−0.024
	Central point loads	M_{75}	−0.125	−0.133	−0.163	−0.144	−0.188	−0.169	−0.139 (28)	−0.178
		M_{76}	−0.125	−0.113	−0.088	−0.106	−0.159	−0.143	−0.111	−0.135
		M_{77}	—	—	—	—	−0.063	−0.081	−0.111	−0.072
		M_{78}*	—	—	—	—	−0.043	−0.032	−0.030	−0.037
	Third-point loads	M_{79}	−0.085	−0.087	−0.089	−0.072	−0.111	−0.100	−0.078 (30)	−0.103
		M_{80}	−0.111	−0.100	−0.078	−0.094	−0.141	−0.127	−0.099	−0.120
		M_{81}	—	—	—	—	−0.055	−0.067	−0.089	−0.063
		M_{82}*	—	—	—	—	−0.038	−0.034	−0.027	−0.032

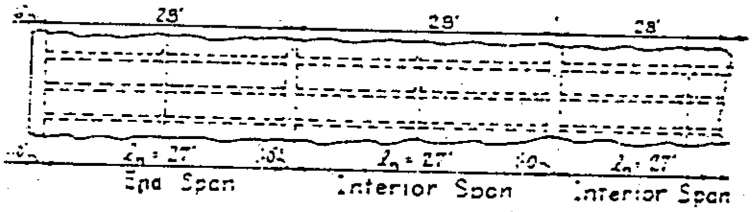


Fig. 8-2 Multiple Span Layout—Ex. 1

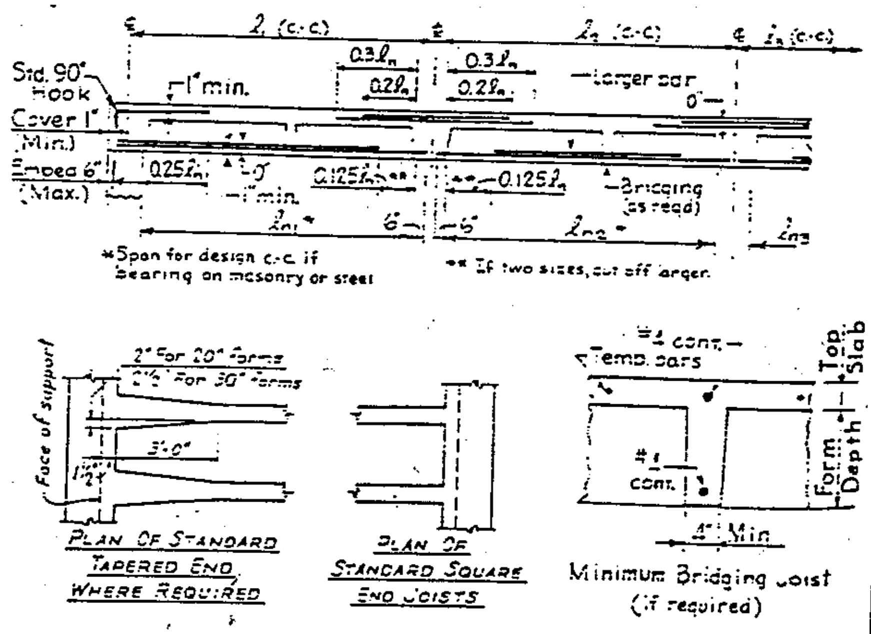


Fig. 8-1 Recommended Bar Details—Multiple Spans

314. **Bending moments in slabs spanning in two directions at right angles with uniformly distributed loads.** The design of solid slabs spanning in two directions at right angles, and of their supporting beams, should be based on one of the three methods given in the following Subclauses *a*, *b* and *c*.

a. METHOD 1. **A purely theoretical analysis based on elastic theory may be made.**

The bending moments in the slabs and beams may be calculated on the assumption that the slabs act as perfectly elastic thin plates, Poisson's ratio being assumed equal to zero. The resistance moment of the slab and beam sections should also be calculated by the commonly employed elastic theory with $m = 15$.

The use of the elastic theory for estimating the bending moments in a slab spanning in two directions is of chief value for slabs with restraint on four sides [see comment on Clause 314 (*b*) (ii)].

b. METHOD 2. **In this method the assessment of bending moments in slabs and beams is based on theoretical analysis amplified and adjusted in the light of experimental data, the resistance moments of the slab and beam sections being calculated by the commonly employed elastic theory, with $m = 15$. The recommendations given in the following Subclauses (i), (ii) and (iii) may be adopted:**

(i) ***Slabs simply supported on four sides.*** **Where, in the case of a simply supported slab, adequate provision is not made to resist torsion at the corners of the slab and to prevent the corners from lifting, the bending moments at mid-span should be assumed to have the values given by the following equations:**

$$M_x = \alpha_x w l_x^2 \qquad (9)$$

$$M_y = \alpha_y w l_x^2 \qquad (10)$$

where M_x and M_y are the bending moments at mid-span on strips of unit width and spans l_x and l_y respectively, w is the total load per unit area, l_y is the length of the longer side, l_x is the length of the shorter side, α_x and α_y are coefficients shown in Table 16.

Table 16. Bending moment coefficients for slabs spanning in two directions at right angles simply supported on four sides

l_y/l_x	1·0	1·1	1·2	1·3	1·4	1·5	1·75	2·0	2·5	3·0
α_x	0·062	0·074	0·084	0·093	0·099	0·104	0·113	0·118	0·122	0·124
α_y	0·062	0·061	0·059	0·055	0·051	0·046	0·037	0·029	0·020	0·014

The bending-moment coefficients given in *Table* 16 are derived from the Grashof-Rankine formulæ which are based on the equality of deflection of the central orthogonal strips of the slab at their intersection. If the slab shown in *Fig.* 11A were subjected to a load w per unit area and were supported only by the longer supports, the bending moments at the centre of a strip of unit width would be $M'_x = \frac{wl_x^2}{8}$, and if it were supported only by the shorter supports the bending moment would be $M'_y = \frac{wl_y^2}{8}$. If now the slab be considered

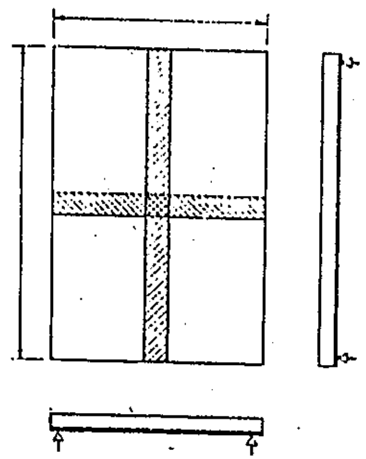


FIG. 11A.

as simply supported on all four edges and the load be assumed to be divided between the two series of strips in such a way that the deflections of the two central strips are the same at the point where they cross, then $w = w_x + w_y$, and $\frac{5}{384}\cdot\frac{w_x l_x^4}{EI_x} = \frac{5}{384}\cdot\frac{w_y l_y^4}{EI_y}$, and, therefore, assuming that $I_x = I_y$, $w_x = w_y\frac{l_y^4}{l_x^4}$ $= w\frac{l_y^4}{l_x^4 + l_y^4}$, and $w_y = w\frac{l_x^4}{l_x^4 + l_y^4}$, from which are obtained the Grashof-Rankine formulæ for the bending moments

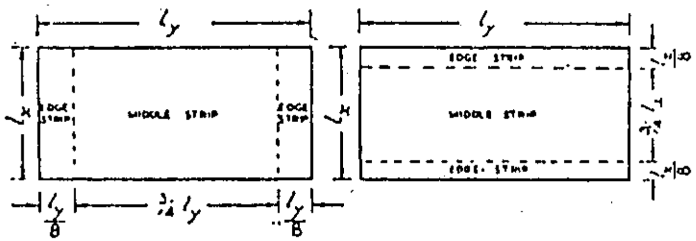
$$M_x'' = w\frac{l_y^4}{l_x^4 + l_y^4}\cdot\frac{l_x^2}{8} = \alpha_x w l_x^2 \text{ and } M_y'' = w\frac{l_x^4}{l_x^4 + l_y^4}\cdot\frac{l_y^2}{8} = \alpha_y w l_x^2$$

in which $\alpha_x = \frac{1}{8}\cdot\frac{(l_y/l_x)^4}{1 + (l_y/l_x)^4}$ and $\alpha_y = \frac{1}{8}\cdot\frac{(l_y/l_x)^2}{1 + (l_y/l_x)^4}$.

The bending moments obtained from this analysis are greater than would actually occur in a simply-supported slab. However, the exact calculation of the moment in such a slab in which the corners can lift is extremely complex. At the same time, neglect of torsion at the corners of the slab makes it desirable in design to use increased bending moments in the span. Such tests as have been carried out have indicated that the additional strength in the span obtained by the use of the coefficients of *Table* 16 influences the behaviour of the slab at the corners sufficiently to enable special corner reinforcement to be omitted.

(ii) *Slabs restrained on four sides.*

1. Where the corners of a slab are prevented from lifting and adequate provision for torsion in accordance with 5 below is made, the bending moments may be assumed to have the values given in 3 below.
2. Slabs are considered as being divided in each direction into middle strips and edge strips as shown in Fig. 2, the middle strip having a width of three-quarters the width of the slab and each edge strip having a width of one-eighth of the width of the slab, except that, for slabs for which the ratio of the sides l_y/l_x exceeds 4·0, the middle strip in the short direction should be taken to have a width of $l_y - l_x$ and each edge strip a width of $l_x/2$.



(a) FOR SPAN l_x (b) FOR SPAN l_y

Fig. 2.—Division of slab into middle and edge strips.

3. The maximum bending moments per unit width in the middle strip of a slab are given by the following equations:

$$M_x = \beta_x w l_x^2 \quad . \quad . \quad . \quad . \quad (11)$$

$$M_y = \beta_y w l_x^2 \quad . \quad . \quad . \quad . \quad (12)$$

where M_x and M_y are the maximum bending moments on strips of unit width in the direction of spans l_x and l_y respectively; w is the total load per unit area; l_y is the length of the longer side; l_x is the length of the shorter side; β_x and β_y are coefficients given in Table 17. By adopting the relationship given in equation (12) it is possible to use a single coefficient for β_y for all ratios of l_y/l_x for each condition of edge support.
4. No reinforcement parallel to the adjacent edges of the slab need be inserted in the edge strips above that required to comply with Clauses 308, 311c and 314b (ii) 5 below.

Table 17. Bending moment coefficients for rectangular panels supported on four sides with provision for torsion at corners

Type of panel and moments considered	Short span coefficients β_x — Values of $\frac{l_y}{l_x}$: 1·0	1·1	1·2	1·3	1·4	1·5	1·75	2·0 or more	Long span coefficients β_y for all values of $\frac{l_y}{l_x}$
Case 1. Interior panels. Negative moment at continuous edge	0·033	0·040	0·045	0·050	0·054	0·059	0·071	0·083	0·033
Positive moment at mid-span	0·025	0·030	0·034	0·038	0·041	0·045	0·053	0·062	0·025
Case 2. One short or long edge discontinuous. Negative moment at continuous edge	0·041	0·047	0·053	0·057	0·061	0·065	0·075	0·085	0·041
Positive moment at mid-span	0·031	0·035	0·040	0·043	0·046	0·049	0·056	0·064	0·031
Case 3. Two adjacent edges discontinuous. Negative moment at continuous edge	0·049	0·056	0·062	0·066	0·070	0·073	0·082	0·090	0·049
Positive moment at mid-span	0·037	0·042	0·047	0·050	0·053	0·055	0·062	0·068	0·037
Case 4. Two short edges discontinuous. Negative moment at continuous edge	0·056	0·061	0·065	0·069	0·071	0·073	0·077	0·080	
Positive moment at mid-span	0·044	0·046	0·049	0·051	0·053	0·055	0·058	0·060	0·044
Case 5. Two long edges discontinuous. Negative moment at continuous edge									0·056
Positive moment at mid-span	0·044	0·053	0·060	0·065	0·068	0·071	0·077	0·080	0·044
Case 6. Three edges discontinuous (one short or long edge continuous). Negative moment at continuous edge	0·058	0·065	0·071	0·077	0·081	0·085	0·092	0·098	0·058
Positive moment at mid-span	0·044	0·049	0·054	0·058	0·061	0·064	0·069	0·074	0·044
Case 7. Four edges discontinuous. Positive moment at mid-span	0·050	0·057	0·062	0·067	0·071	0·075	0·081	0·083	0·050

5. Torsion reinforcement should be provided at the corners of a slab except at corners contained by edges over both of which the slab is continuous.

At corners contained by edges over neither of which the slab is continuous, top and bottom reinforcement should be provided for torsion at the corners of the slabs. Both top and bottom reinforcement should consist of two layers of bars placed parallel to the sides of the slab and extending in these directions for a distance of one-fifth of the shorter span. The area of the bars in each of the four layers, per unit width of the slab, should be three-quarters of the area required for the maximum positive moment in the slab.

At corners contained by edges over only one of which the slab is continuous, the torsional reinforcement may be reduced to one-half of that required by the preceding paragraph.

Any reinforcement provided for the purpose of complying with other clauses of this Code may be included as part of the reinforcement required to comply with this clause.

6. Where a slab ends and there is monolithic connection between the slab and the supporting beam or wall, provision should be made for the negative moments that may occur in the slab at such support. The negative moment to be assumed in these cases depends on the degree of fixity afforded to the edge of the slab, but for general purposes it may be taken as two-thirds of the moment given in Table 17 for the mid-span of the slab.

The rules given in this clause are based on American regulations, which were derived from mathematical analyses by Professor Westergaard supplemented by test data. The effects of redistribution of bending moment have been partially allowed for and this has enabled the number of coefficients in *Table 17* to be reduced to a minimum.

The simplification of the design procedure for a wide range of conditions at the edges is coupled with the requirement that reinforcement should be provided at corners where one or both edges are discontinuous. For most practical cases of slab panels in buildings, even where one or more sides are discontinuous, the form of edge support is such as to provide a certain amount of fixity throughout its length. This partial fixity may be realised by the slab being monolithic with its supporting beam and suitably reinforced, by the slab entering a brick wall and being held down by the weight above, or by other means. Such practical fixity (provided that resistance to negative bending moment can be developed) has the effect of reducing the torsion at the corners, and proportionately the necessity for reinforcement at the corners. This effect cannot apparently be allowed for if the simplified coefficients in *Table 17* are adopted, but can be taken into account if the bending moments are calculated according to the theory of the elastic thin plate, as permitted by Clause 314 (*a*).

When a slab is restrained along one or more of its edges, the mathematical investigation becomes involved and in many cases has not yet been worked out completely. By making certain simplifications, however, it is possible, with a reasonable degree of accuracy, to predict the bending moments in such panels.

Dr Marcus's Method.—The approximate method of slab design developed by Dr H. Marcus in Germany,* and briefly outlined here, is similar in derivation

* 'Die Theorie Elastischer Gewebe . . .' (Julius Springer, Berlin).

to the Grashof-Rankine formulæ (see page 71) but introduces a simple correction to allow for restraint at corners and for the assistance given by torsion. It has been shown that the bending moments obtained in this simple manner vary by only 1 per cent. or 2 per cent. from those which have been obtained from more rigorous analyses based on the elastic plate theory.

The correction factors to the Grashof-Rankine formulæ for slabs simply supported on all four edges, given by Dr Marcus, are $C_x = 1 - \frac{5}{6}\cdot\frac{M''_x}{M'_y}$ and $C_y = 1 - \frac{5}{6}\cdot\frac{M''_y}{M'_x}$, and the moments to be used in design at mid-span are therefore $M_x = C_x M''_x$ and $M_y = C_y M''_y$, which may be expressed as follows

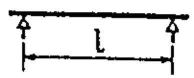
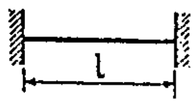
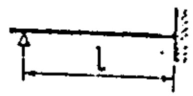
$$M_x = \left(1 - \frac{5}{6}\cdot\frac{r^2}{1+r^4}\right)M''_x$$

$$M_y = \left(1 - \frac{5}{6}\cdot\frac{r^2}{1+r^4}\right)M''_y$$

where $r = \frac{l_y}{l_x}$.

The procedure for slabs with any or all of the sides fixed is the same, the values of M''_x and M''_y being obtained from consideration of the strips according to the particular conditions of fixity obtaining as shown in *Table 11A*.

Table 11A.

			
Deflection	$\frac{5}{384}\frac{w_x l_x^4}{EI}$	$\frac{1}{384}\frac{w_x l_x^4}{EI}$	$\frac{2}{384}\frac{w_x l_x^4}{EI}$
Moment M''_x	$\frac{1}{8}w_x l_x^2$	$\frac{1}{24}w_x l_x^2$	$\frac{9}{128}w_x l_x^2$

It should be noted that the deflections are taken at mid-span but that the values of M''_x and M''_y are maxima for the span under consideration and are not necessarily at mid-span.

The six possible conditions of support, together with the appropriate values of the proportion of the load carried in each direction, and the expressions for C_x and C_y in terms of span ratios, are given in *Table 12A*.

Moments at Supports.—According to the German regulations, in which Dr Marcus's method is adopted, the bending moments at the supports are calculated as for slabs spanning in one direction only and the load used is the same as that employed for the calculations of the bending moments on the span. Thus, when two opposite edges are fixed, the average bending moment along these edges would be $-\frac{1}{12}w_x l_x^2$ or $-\frac{1}{12}w_y l_y^2$. When one edge is free and the opposite edge is fixed the bending moment would be $-\frac{1}{8}w_x l_x^2$ or $-\frac{1}{8}w_y l_y^2$. These moments

TABLE 12A.

$r = \frac{l_y}{l_x}$ Conditions of Support	Proportion of Load in each Direction: In direction l_x $\frac{w_x}{w}$	In direction l_y $\frac{w_y}{w} = 1 - \frac{w_x}{w}$	Uncorrected Span Moments: M''_x	M''_y	Values of Coefficients for Span Moments: C_x $(M_x = C_x M''_x)$	C_y $(M_y = C_y M''_y)$
I	$\frac{r^4}{1+r^4}$	$\frac{1}{1+r^4}$	$\frac{1}{8} w_x l_x^2$	$\frac{1}{8} w_y l_y^2$	$1 - \frac{5}{6} \frac{r^2}{1+r^4}$	$1 - \frac{5}{6} \frac{r^2}{1+r^4}$
II	$\frac{5r^4}{2+5r^4}$	$\frac{2}{2+5r^4}$	$\frac{9}{128} w_x l_x^2$	$\frac{1}{8} w_y l_y^2$	$1 - \frac{75}{32} \frac{r^2}{2+5r^4}$	$1 - \frac{5}{3} \frac{r^2}{2+5r^4}$
III	$\frac{5r^4}{1+5r^4}$	$\frac{1}{1+5r^4}$	$\frac{1}{24} w_x l_x^2$	$\frac{1}{8} w_y l_y^2$	$1 - \frac{25}{18} \frac{r^2}{1+5r^4}$	$1 - \frac{5}{6} \frac{r^2}{1+5r^4}$
IV	$\frac{r^4}{1+r^4}$	$\frac{1}{1+r^4}$	$\frac{9}{128} w_x l_x^2$	$\frac{9}{128} w_y l_y^2$	$1 - \frac{15}{32} \frac{r^2}{1+r^4}$	$1 - \frac{15}{32} \frac{r^2}{1+r^4}$
V	$\frac{2r^4}{1+2r^4}$	$\frac{1}{1+2r^4}$	$\frac{1}{24} w_x l_x^2$	$\frac{9}{128} w_y l_y^2$	$1 - \frac{5}{9} \frac{r^2}{1+2r^4}$	$1 - \frac{15}{32} \frac{r^2}{1+2r^4}$
VI	$\frac{r^4}{1+r^4}$	$\frac{1}{1+r^4}$	$\frac{1}{24} w_x l_x^2$	$\frac{1}{24} w_y l_y^2$	$1 - \frac{5}{18} \frac{r^2}{1+r^4}$	$1 - \frac{5}{18} \frac{r^2}{1+r^4}$

are not the greatest that occur. In the case of a slab fixed on all sides the maximum value may be obtained approximately by dividing the average values by C_x (*Table* 12A, Case VI). The greatest value of this correction occurs for a square slab, and is about 15 per cent. Since, however, these bending moments occur over only a short length of the support, and the bending moments are less elsewhere along the support, it is quite safe to use the average value.

In the case of the shorter sides of long panels the bending moments at the support calculated in this way are very small. The same remark applies to the bending moments on the longitudinal spans of such panels, and it is considered desirable that increased values be taken in both these cases.

In *Tables* 13A to 23A the bending-moment factors are adjusted so that the moments $\left(\frac{wl_y^2}{K_y}\right)$ on the longitudinal span are in no case less than $\frac{wl_x^2}{50}$ and the moments $\left(-\frac{wl_y^2}{}\right)$ at the supports in the same direction are in no case less than

$-\frac{wl_x^2}{40}$. The adjusted factors are given in italics and are recommended for use.

MOMENTS IN CONTINUOUS PANELS.—A method of estimating the bending moments produced on various systems of equal and continuous panels, based on the work of Dr Marcus, is given by Professor B. Loser.* In this method calculations of the bending moments are made for two conditions of loading (*Fig.* 12A), namely

(*b*) the dead load of a panel together with one-half the live load are considered to act over all the panels, and
(*c*) one-half the live load is considered to act alternately upwards and downwards in consecutive panels.

The sums of the mid-span moments for the two conditions are then equal to the values that must be taken into account in design, that is, dead load on all panels and live load on alternate panels as in *Fig.* 12A (*a*). This method has the advantage that the bending moments for the two auxiliary conditions of loading can be obtained on the following assumptions: For the uniform load throughout (*b*), the panels may be considered as fixed at interior supports and freely supported at the boundary supports of the system; for the alternate upward and downward loading (*c*), each panel may be considered as simply supported.

The method of computing the bending moments at the supports is as explained previously. Where there are only two panels in the direction under consideration, the bending-moment coefficient for the intermediate support is assumed to be one-eighth. Where there are three or more panels the coefficient for the first interior support is assumed to be one-tenth, and the coefficient for other interior supports is assumed to be one-twelfth.

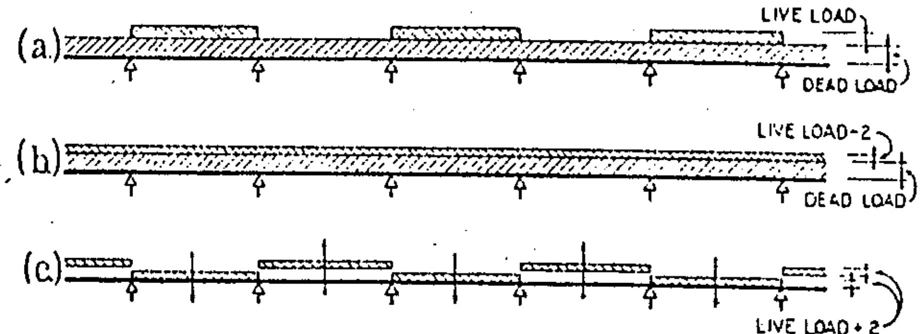


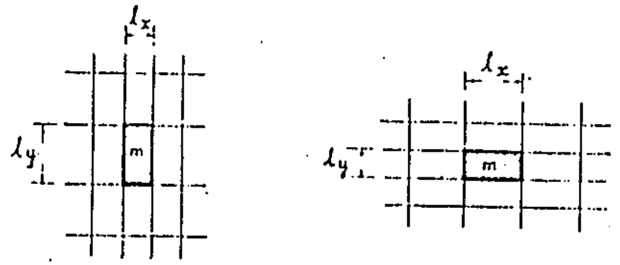
FIG. 12A.

Tables 13A to 23A give bending-moment factors for various arrangements of rectangular panels continuous with panels of equal proportions as shown. The sketch at the head of each table shows the particular panel and supports to which the factors in the table relate.

No reduction (Clause 312) should be made to the bending moments in two-way slabs, whether *Table* 17 or *Tables* 13A to 23A are used, since both methods are based upon approximate factors in which suitable allowances for redistribution of bending moments have been made.

* 'Momentenausgleichsverfahren' (W. Ernst & Sohn, Berlin.)

TABLE 23A.—SLAB PANELS.
BENDING MOMENT FACTORS FOR DISTRIBUTED LOAD.



FULLY CONTINUOUS PANEL.

SHAPE OF PANEL l_y/l_x	INTERIOR SPAN l_x K_x RATIO DEAD : LIVE LOAD 1 : 1	1 : 2	1 : 4	INTERIOR SUPPORTS $-S_x$ DEAD : LIVE LOAD ALL RATIOS	INTERIOR SPAN l_y K_y RATIO DEAD : LIVE LOAD 1 : 1	1 : 2	1 : 4	INTERIOR SUPPORTS $-S_y$ DEAD : LIVE LOAD ALL RATIOS
2·00	19·5	17·9	16·7	12·7	*200*	*200*	*200*	*160*
1·80		19·0		13·1		*162*		*130*
1·60		20·9		13·8		*128*		90·6
1·50		22·3		14·4		112		72·8
1·40		24·1		15·1		92·6		58·1
1·30		26·5		16·2		75·8		46·3
1·20		29·9		17·8		62·0		36·9
1·10		34·6		20·2		50·7		29·6
1·00	44·3	41·5	39·4	24·0	44·3	41·5	39·4	24·0
0·95		46·1		26·7		37·5		21·8
0·90		52·0		30·3		34·0		19·9
0·85		58·9		35·0		30·9		18·2
0·80		68·3		41·3		28·0		16·9
0·75		81·1		49·9		25·6		15·8
0·70		98·0		62·0		23·5		14·9
0·60		*139*		104		20·2		13·5
0·50	*200*	*200*	*200*	*160*	19·5	17·9	16·7	12·7

MOMENT ON SPAN $M_x = \frac{wl_x^2}{K_x}$ MOMENT ON SPAN $M_y = \frac{wl_x^2}{K_y}$

MOMENT AT SUPPORT $M_x = -\frac{wl_x^2}{S_x}$ MOMENT AT SUPPORT $M_y = -\frac{wl_x^2}{S_y}$

(iii) *Loads on supporting beams.* The loads on the supporting beams may be assumed to be in accordance with *Fig. 3.*

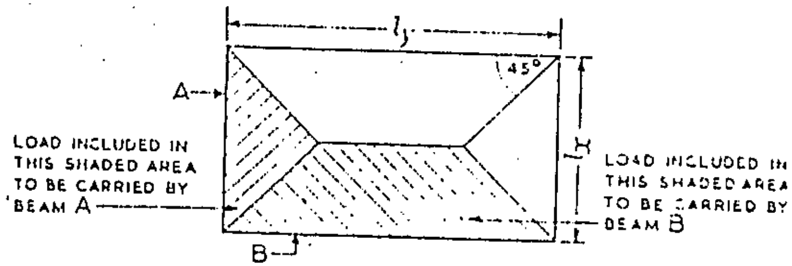


Fig. 3.—Diagram showing the load carried by supporting beams.

The geometrical method of estimating the loads carried by the beams given by Clause 314 (*b*) (iii) has been commonly used in this country. The total loads on the short and long spans due to one loaded panel are given by

$$\text{Load on short span beam} = W_x = \frac{wl_x^2}{4}.$$

$$\text{Load on long span beam} = W_l = \frac{wl_xl_y}{2} - W_x.$$

The corresponding bending moments in the beams may be determined with sufficient accuracy by assuming that the loading is equivalent to a uniform load per unit length of the beam of the following amounts:

$$\text{On the short span, } \frac{wl_x}{3}.$$

$$\text{On the long span, } \frac{wl_x}{6}\left[3 - \left(\frac{l_x}{l_y}\right)^2\right].$$

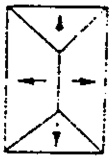
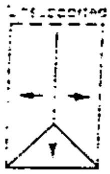
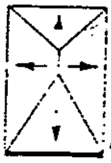
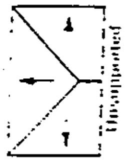
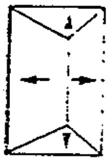
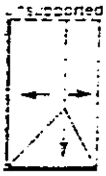
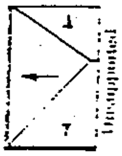
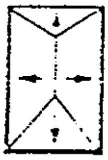
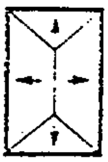
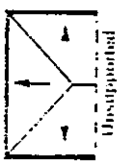
The shearing force should, however, be determined from the actual loads obtained from *Fig. 3.*

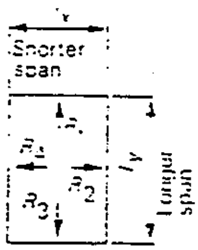
c. METHOD 3. This method is based on the load-factor method of design. The slabs and beams may be designed to have a load factor generally of 1·8; in the calculations of the ultimate strength, however, the cube strength of the concrete should be taken as only two-thirds of the actual cube strength when designed concrete mixes are used, or three-fifths of the actual cube strength when nominal mixes are used. This requirement should be complied with in the following way. The ultimate bending moments to be allowed for should be deduced from analysis in which the load is 1·8 times the working (dead and imposed) load and due regard is given to redistribution of moments that would occur before failure of the slab or beam, by the use of Johansen's yield-line theory or other acceptable method; the resistance moments of the slab and beam sections should be calculated in accordance with the recommendations of Clause 306; and these resistance moments should be equal to at least 55 per cent of the ultimate bending moments at failure.

The design of beams or slabs involves (i) the determination of the bending moments throughout the system, and (ii) the determination of suitable sections to resist those moments. The load-factor method of estimating resistance

Two-way slabs: Rectangular panels: Loads on beams

Table 54

Panels supported along four edges	Panels unsupported along one (or two) edges
 $k > 1$: $R_1 = R_3 = \frac{1}{4} w l_x^2$; $R_2 = R_4 = \frac{1}{2}(k - \frac{1}{2}) w l_x^2$. $k = 1$: $R_1 = R_2 = R_3 = R_4 = \frac{1}{4} w l_x^2$	 (Unsupported) $R_1 = 0$; $R_2 = R_4 = \frac{1}{2}(k - \frac{1}{4}) w l_x^2$; $R_3 = \frac{1}{4} w l_x^2$
 $k > 1\frac{1}{3}$: $R_1 = \frac{1}{4} w l_x^2$ (min.); $R_2 = R_4 = \frac{1}{2}(k - \frac{2}{3}) w l_x^2$; $R_3 = \frac{5}{12} w l_x^2$ (max.). $k \leq 1\frac{1}{3}$: $R_1 = \frac{3}{5} R_3$ approx. (min.); $R_2 = R_4 = \frac{3}{16} k^2 w l_x^2$; $R_3 = \frac{5}{8} k(1 - \frac{3}{8} k) w l_x^2$ approx. (max.)	 (Unsupported) $k > 2$: $R_1 = R_3 = \frac{1}{2} k(1 - \frac{1}{4} k) w l_x^2$; $R_2 = 0$; $R_4 = \frac{1}{4} k^2 w l_x^2$
 $R_1 = R_3 = \frac{3}{16} w l_x^2$; $R_2 = \frac{3}{5} R_4$ (min.); $R_4 = \frac{5}{8}(k - \frac{3}{8}) w l_x^2$ (max.)	 (Unsupported) $R_1 = R_2 = 0$; $R_3 = \frac{1}{2} w l_x^2$; $R_4 = (k - \frac{1}{2}) w l_x^2$
 $R_1 = \frac{3}{16} w l_x^2$ (min.); $R_2 = \frac{3}{5} R_4$ (min.); $R_3 = \frac{5}{16} w l_x^2$ (max.); $R_4 = \frac{5}{8}(k - \frac{1}{2}) w l_x^2$ (max.)	 (Unsupported) $R_1 = 0$; $R_2 = \frac{3}{5} R_4$ (min.); $R_3 = \frac{5}{16} w l_x^2$; $R_4 = \frac{5}{8}(k - \frac{5}{16}) w l_x^2$ (max.)
 $k > 1\frac{1}{2}$: $R_1 = R_3 = \frac{5}{16} w l_x^2$; $R_2 = \frac{3}{5} R_4$ (min.); $R_4 = \frac{5}{8}(k - \frac{5}{8}) w l_x^2$ (min.). $k \leq 1\frac{1}{2}$: $R_1 = R_3 = \frac{1}{2} k(1 - \frac{3}{8} k) w l_x^2$; $R_2 = \frac{5}{32} k^2 w l_x^2$ (min.); $R_4 = \frac{1}{4} k^2 w l_x^2$ (max.)	 (Unsupported) $k < 1\frac{2}{3}$: $R_1 = \frac{3}{5} R_3$ (min.); $R_2 = 0$; $R_3 = \frac{1}{2} k(1 - \frac{3}{10} k) w l_x^2$ (max.); $R_4 = \frac{3}{10} k^2 w l_x^2$. $k \geq 1\frac{2}{3}$: $R_1 = \frac{3}{8} w l_x^2$ (min.); $R_2 = 0$; $R_3 = \frac{1}{2} w l_x^2$; $R_4 = (k - \frac{7}{8}) w l_x^2$ (max.)
 $R_1 = \frac{3}{16} w l_x^2$ (min.); $R_2 = R_4 = \frac{1}{2}(k - \frac{1}{2}) w l_x^2$; $R_3 = \frac{5}{16} w l_x^2$ (max.)	 (Unsupported) $R_1 = 0$; $R_2 = R_4 = \frac{1}{2}(k - \frac{1}{4}) w l_x^2$; $R_3 = \frac{1}{4} w l_x^2$
 $k > 1$: $R_1 = R_3 = \frac{1}{4} w l_x^2$; $R_2 = R_4 = \frac{1}{2}(k - \frac{1}{2}) w l_x^2$. $k = 1$: $R_1 = R_2 = R_3 = R_4 = \frac{1}{4} w l_x^2$	 (Unsupported) $k > 2$: $R_1 = R_3 = \frac{1}{2} k(1 - \frac{1}{4} k) w l_x^2$; $R_2 = 0$; $R_4 = \frac{1}{4} k^2 w l_x^2$



$$k = \frac{l_y}{l_x} = \frac{\text{Longer span}}{\text{Shorter span}}$$

w = Intensity of uniformly-distributed service load per unit area

If analysis due to ultimate loads is undertaken, substitute n for w in appropriate formulae.

R_1, R_2, R_3, R_4 = total load carried by each support of panel

Condition of supports
- – – – – = No support
- ——— = Freely supported
- ▬▬▬ = Continuity or fixity

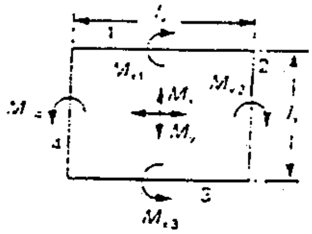
Loads marked (min.) apply if panel is entirely freely supported along edge indicated: if partially restrained, load will be slightly greater than given and load marked (max.) on opposite edge will be correspondingly reduced.

Two-way slabs: Rectangular panels: Uniform load: Elastic analyses

Ratio of spans k	Condition along all edges					
	Free. Corners not held down		Free or fixed. Corners held down		Free (Dr Marcus)	Fixed (Dr Marcus)
	α_{x2}	α_{y2}	α_{x3}	α_{y3}	ξ_1	ξ_2
1.0	0.500	0.500	0.300	0.300	0.583	0.361
1.05	0.549	0.451	0.330	0.271	0.585	0.362
1.1	0.594	0.406	0.360	0.246	0.591	0.364
1.15	0.636	0.364	0.389	0.222	0.599	0.366
1.2	0.675	0.325	0.418	0.202	0.610	0.370
1.25	0.709	0.291	0.446	0.183	0.622	0.374
1.3	0.741	0.259	0.474	0.166	0.635	0.378
1.4	0.793	0.207	0.526	0.137	0.663	0.388
1.5	0.835	0.165	0.575	0.114	0.691	0.397
1.6	0.868	0.132	0.621	0.095	0.718	0.906
1.75	0.904	0.096	0.682	0.073	0.754	0.918
2.0	0.941	0.059	0.763	0.048	0.804	0.935
2.5	0.975	0.025	0.892	0.023	0.870	0.957
3.0	0.988	0.012	0.972	0.012	0.909	0.970

Ratio of spans $k = \frac{\text{long span}}{\text{short span}} = \frac{l_y}{l_x}$

Uniform service load = w per unit area



Freely supported along all four edges:

Corners not held down:

$M_x = -\alpha_{x2}(\frac{1}{8}wl_x^2)$; $M_y = -\alpha_{y2}(\frac{1}{8}wl_x^2) = M_x/k^2$

Corners held down:

$M_x = -\alpha_{x3}(\frac{1}{8}wl_x^2)$; $M_y = -\alpha_{y3}(\frac{1}{8}wl_x^2)$

Corners held down (Dr Marcus's method):

$M_x = -\xi_1\alpha_{x2}(\frac{1}{8}wl_x^2)$; $M_y = -\xi_1\alpha_{y2}(\frac{1}{8}wl_x^2) = M_x/k^2$

Fixed along all four edges:

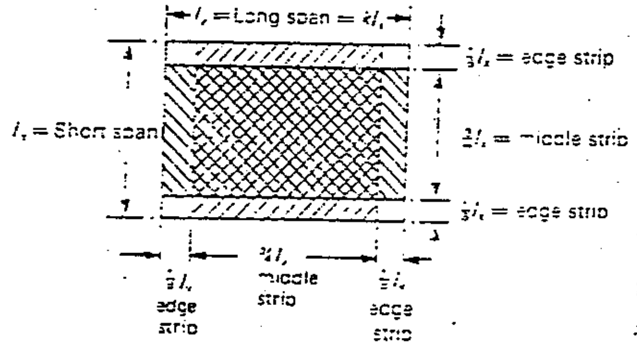
Corners held down (Dr Marcus's method):

$M_x = -\xi_2\alpha_{x2}(\frac{1}{24}wl_x^2)$; $M_y = -\xi_2\alpha_{y2}(\frac{1}{24}wl_x^2) = M_x/k^2$

$M_{x1} = M_{x3} = -\alpha_{x2}(\frac{1}{12}wl_x^2)$;

$M_{y2} = M_{y4} = -\alpha_{y2}(\frac{1}{12}wl_x^2) = M_{x1}/k^2$

Continuity (or fixity) along one or more edges. (CP110 and CP114)



Conditions: Corners held down

Torsional resistance provided

No main reinforcement required in edge strips to resist bending moment parallel to edges of panel.

CP110 requirements:

If $k > 2$, the adjoining arrangement applies.

If $k > 2$, the panel should be designed to span in one direction only.

CP114 requirements:

If $k > 4$, the adjoining arrangement applies.

If $k > 4$, width of middle strip $= l_y - l_x$ and width of edge strip $= \frac{1}{2}l_x$.

n = total ultimate load w = total service load

Bending moments on middle strip	At midspan		At continuous edge		At discontinuous edge (Slab monolithic with support)	
	CP110	CP114	CP110	CP114	CP110	CP114
Short span	$\beta_{x1}nl_x^2$	$\beta_{x2}wl_x^2$	$-\beta_{x1}'nl_x^2$	$-\beta_{x2}'wl_x^2$	$-\frac{1}{2}\beta_{x1}nl_x^2$	$-\frac{2}{3}\beta_{x2}wl_x^2$
Long span	$\beta_{y1}nl_x^2$	$\beta_{y2}wl_x^2$	$-\beta_{y1}'nl_x^2$	$-\beta_{y2}'wl_x^2$	$-\frac{1}{2}\beta_{y1}nl_x^2$	$-\frac{2}{3}\beta_{y2}wl_x^2$

Values given by CP110 expressions are *ultimate* moments; values given by CP114 expressions are *service* moments.
For values of β_{x1}, β_{x1}', β_{y1} and β_{y1}' see Table 51; for values of β_{x2}, β_{x2}', β_{y2} and β_{y2}' see Table 52.

Main reinforcement (CP110 only)

Without torsional restraint

$\nless \frac{1}{2}A_s$ should extend to support; remainder to $0.1l$ from support.

With torsional restraint

Span: $\nless \frac{1}{2}A_s$ should extend to 50mm from discontinuous edge; remainder to $0.15l$ from edge.
$\nless \frac{1}{2}A_s$ should extend to $0.15l$ from continuous edge; remainder to $0.25l$ from edge.

Support: $\nless \frac{1}{2}A_s$ should extend to $0.3l$ from continuous edge; remainder to $0.15l$ from edge.
At discontinuous support $A_{s\,reqd}$ depends on fixity; generally $\frac{1}{2}A_s$ provided at midspan extending $0.1l$ into span will suffice.

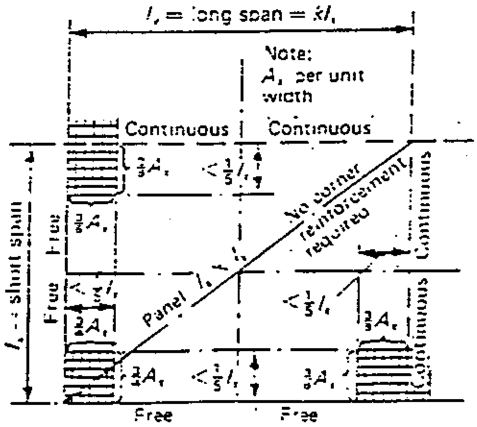
Corner reinforcement for torsional resistance

(CP110 and CP114) (see adjoining diagram)

A_x and A_y = cross-sectional area (per unit width) of reinforcement for positive B.M. at midspan of short and long spans respectively.

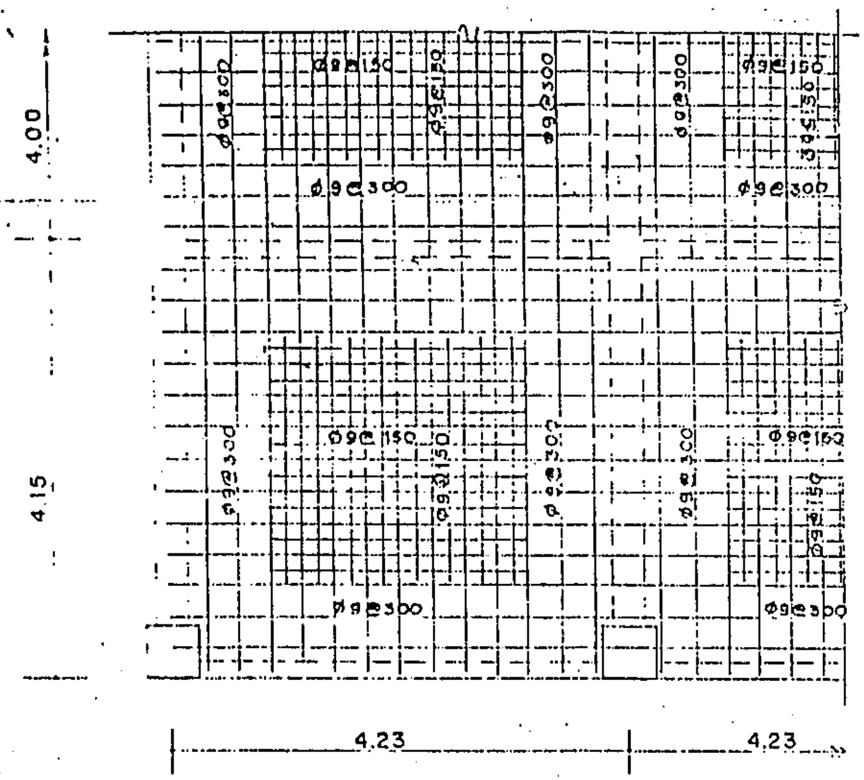
$\frac{3}{4}A_x$ and $\frac{3}{8}A_x$ = cross-sectional area of corner reinforcement in each of two layers (one near top face of slab; one near bottom face), sq. in per ft or sq. mm per m

If $A_y > A_x$ substitute $\frac{3}{4}A_y$ and $\frac{3}{8}A_y$ for $\frac{3}{4}A_x$ and $\frac{3}{8}A_x$

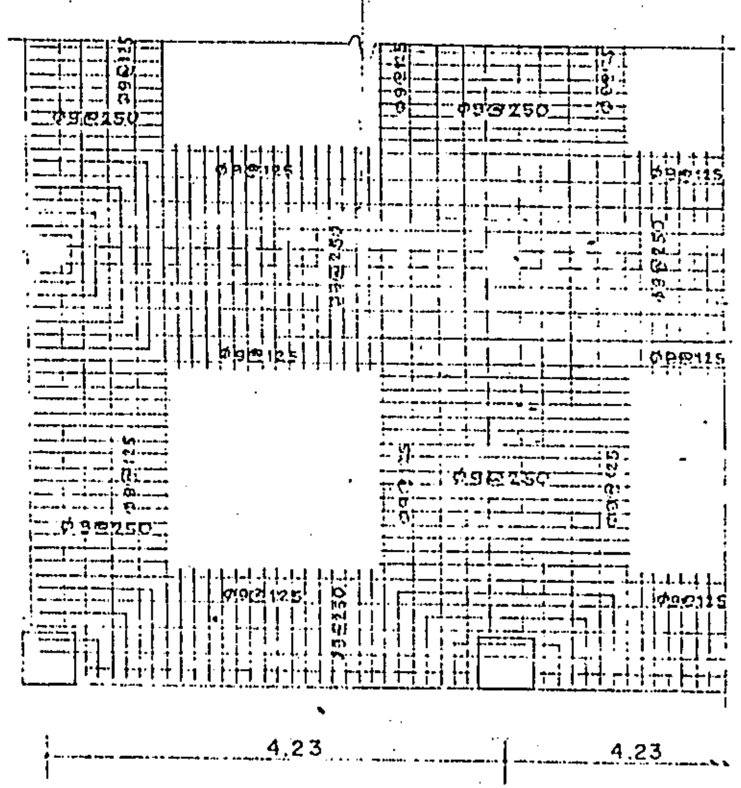


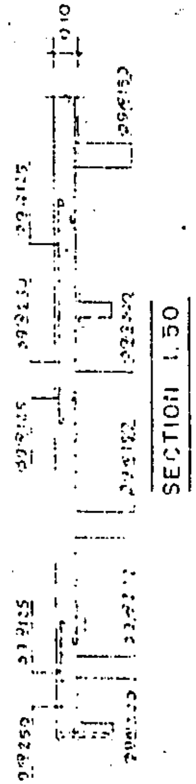
SLAB REINFORCEMENT DETAIL A | 1 : 50

BOTTOM REINFORCEMENT

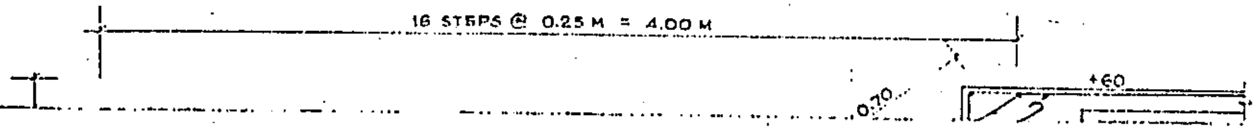


TOP REINFORCEMENT



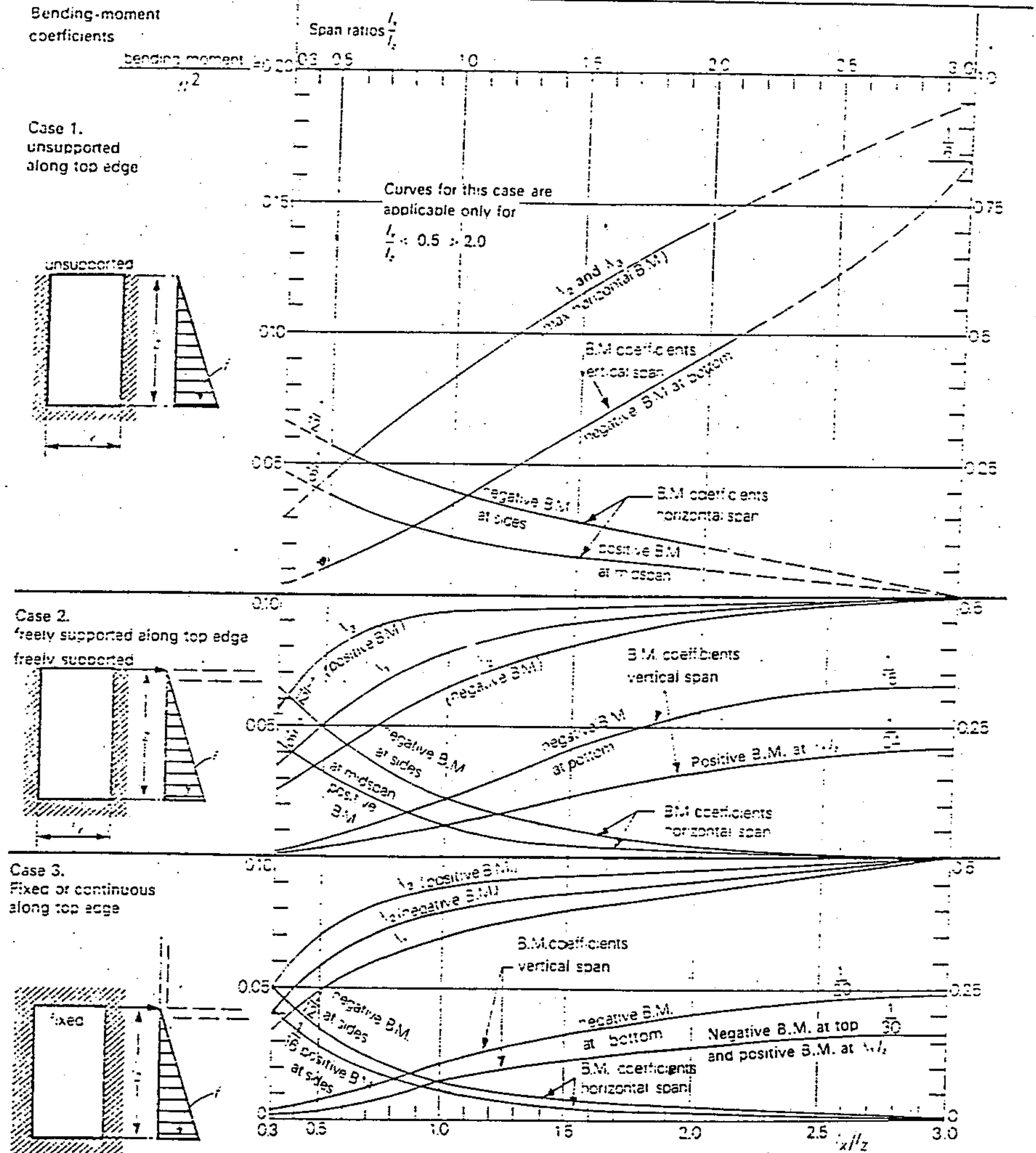


SLAB REINFORCEMENT DETAIL B | 1 : 50

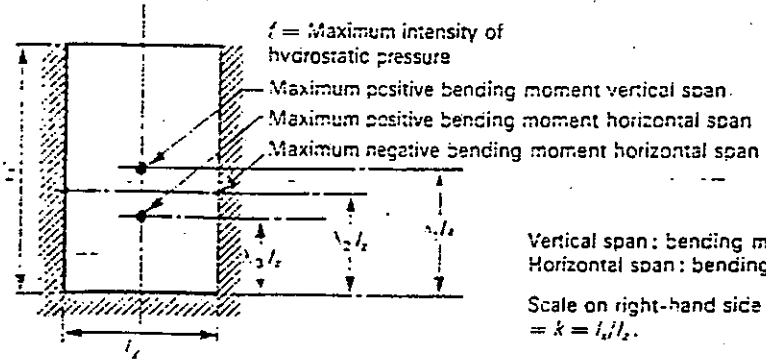


Two-way slabs: Rectangular panels: Triangularly-distributed loads

Table 53



Positions of maximum bending moments



Notes

Panels fixed or continuous along bottom edge and both vertical sides; condition along top edge as indicated.

Fractions thus '$\frac{1}{12}$' indicate coefficients to which curves are asymptotic or to which coefficients approach as span ratio l_x/l_z approaches zero or infinity.

Vertical span: bending moment = (coefficient) fl_z^2
Horizontal span: bending moment = (coefficient) fl_x^2

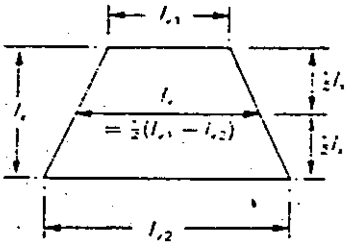
Scale on right-hand side is for values of λ_1, λ_2 and λ_3. Ratio of spans $= k = l_x/l_z$.

Two-way slabs: Elastic analysis of non-rectangular panels

Table 57

Trapezium

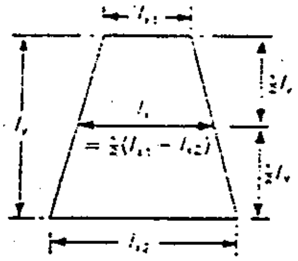
l_x [illegible] l_y



Calculate bending moments as for rectangular panel with

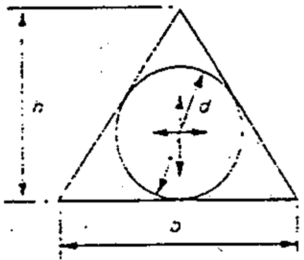
$$k = \frac{l_y}{l_x}$$

$l_x < l_y$



If l_{y1} is small compared with l_{y2} or l_{x1} is small compared with l_{x2}: Apply rules for triangular panel

Isosceles triangle



$$d = \text{diameter of inscribed circle} = \frac{2bh}{b + \sqrt{b^2 + 4h^2}}$$

Freely-supported along all edges (corners restrained).

Bending moment (in two directions at centre of circle) $= -\dfrac{wd^2}{16}$

Continuous along all sides.

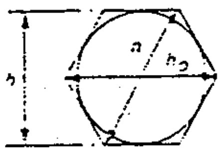
Bending moment (in two directions at centre of circle) $= -\dfrac{wd^2}{30}$

Bending moment (at sides) $= -\dfrac{wh^2}{30}$

w = intensity of uniformly-distributed load (or intensity of pressure at centre of circle if pressure varies uniformly.)

Regular polygon

Five or more sides.



h = diameter of inscribed circle = distance across flats.

h_0 = diameter of circumscribed circle

= distance across corners.

$h_1 = \frac{1}{2}(h + h_0) = 1.08h$ for hexagon
$1.04h$ for octagon

Calculate bending moments as for circle of diameter h_1.

Circle (diameter = h)

Concentric concentrated load F

Freely-supported around edge.

M_{tot} = total positive bending moment across diameter $= \dfrac{Fh}{2\pi}\left(1 - \dfrac{2d}{3h}\right)$

M_m = mean positive bending moment across diameter $= M_{tot}/h$

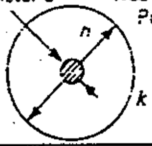
M_{max} = maximum positive bending moment at centre $= 1.5\ M_{tot}$ (approximate)

Restrained around edge. Negative bending moment at edge $< \frac{1}{3}M_m$

Positive bending moment: Average across diameter $= kM_m$

Maximum at centre $= kM_{max}$

Load F on area of diameter d



$$k = \frac{2}{15}\left(\frac{d}{h}\right) + \frac{2}{3}$$

Notes

Reinforcement to resist positive bending moments to be provided in two directions mutually at right-angles.

Uniformly-distributed load w over entire panel

Condition at circumference	Positive bending moment: Average across diameter	Positive bending moment: Alternative maximum at centre	Negative bending moment around edge
Freely supported	$\dfrac{wh^2}{24}$	$\dfrac{wh^2}{16}$	—
Partially restrained (continuous)	$\dfrac{wh^2}{48}$	$\dfrac{wh^2}{32}$	$\dfrac{wh^2}{32}$
Fixed	$\dfrac{wh^2}{48}$ (minimum)	$\dfrac{wh^2}{48}$	$\dfrac{wh^2}{24}$

Reinforcement to resist negative bending moment around edge to be radial (or equivalent). If maximum positive bending moment is designed for, reinforcement (or thickness of slab) can be reduced towards edge.

For notes on this table, see pages 32 and 33 and page 232.